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SYLLABUS

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FIRST TERM

March - May

UNIT VI: PROBABILITY

Probability:
Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution.

UNIT I. RELATIONS AND FUNCTIONS

Relations and Functions:
Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function, Binary operations.

Inverse Trigonometric Functions:
Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

UNIT III: CALCULUS

Continuity and Differentiability:
Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivative. Logarithmic differentiation. Derivative of
functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

Applications of Derivatives:
Applications of derivatives: rate of change, increasing/decreasing functions, tangents&normals, approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

July

Integrals:
Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type

\[ \int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{a^2 - x^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{(px + q)dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{(px + q)dx}{\sqrt{ax^2 + bx + c}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{\sqrt{x^2 \pm a^2}} \]
to be evaluated.

August

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

Applications of the Integrals:
Applications in finding the area under simple curves, especially lines, areas of circles/Parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

UNIT-V: LINEAR PROGRAMMING

Linear Programming:
Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).
SECOND TERM

October

Differential Equations:
Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + p(x)y = q(x),$$
where p(x) and q(x) are functions of x.

UNIT-II: ALGEBRA

Matrices:
Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutative of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Determinants:
Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.
UNIT-IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY

Vectors:

Three - dimensional Geometry:
Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.
Assignment No. 1

Relations, Functions and Binary Operations

Note: Q1-10 are very short answer type questions

1. If the function $f : A \rightarrow A$ be defined by $f(x) = x^2 + 1$, find $f^{-1}(3)$.

2. If * is defined on $Q \setminus \{1\}$ by $a*b = a+b-ab$, for all $a,b \in Q \setminus \{1\}$, find the identity element in $Q \setminus \{1\}$.

3. Let $A = \{2,3,4,5\}$. Define a relation on $A$ which is reflexive and symmetric but not transitive.

4. If $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined by $f(x) = 8x^3$, $g(x) = x^\frac{1}{3}$, then find $fog$ and $gof$.

5. If $f(x) = (a-x^n)^\frac{1}{n}$, then find $(f \circ f)(x)$

6. Let $f(x) = x^2 - 2$; $g(x) = x + 2$, $x \in R$, find $(g \circ f)(1)$

7. Let * be a binary operation defined by $a*b = 3a + 4b - 2$, then find $2*(3*2)$

8. If $f(x)$ is an invertible function, find the inverse of $f(x) = \frac{3x - 2}{5}$

9. If $f(x) = \frac{x-1}{x+1}$, find $(fof^{-1})(2)$

10. Let $f$ be the greatest integer function and $g$ be the absolute value function, find the value of $(gof)\left(\frac{5}{3}\right) - (fog)\left(\frac{-5}{3}\right)$.

11. Let $A = Q \times Q$ and * be a binary operation on $A$ defined by $(a,b) * (c,d) = (ac+b+ad)$ for $(a,b),(c,d) \in A$. Find the identity element in $A$ and the invertible elements of $A$.

12. Show that $f : R \setminus \{-1\} \rightarrow R \setminus \{1\}$ given by $f(x) = \frac{x}{x+1}$ is invertible. Also find its inverse.

13. Prove that the relation $R$ on the set $N \times N$ defined by $(a,b) R (c,d) \iff a+d = b+c \land (a,b),(c,d) \in N \times N$ is an equivalence relation.
14. Let $N$ be the set of all natural numbers and $R$ be a relation on $N \times N$, defined as 
\[(a, b) R (c, d) \iff ad = bc, \text{ for all } (a, b), (c, d) \in N \times N.\] Show that “$R$” is an equivalence relation.

15. If $f : R \to R$ is a function defined by $f(x) = x^3 + 27 \ \forall \ x \in R$. Show that $f$ is bijective and hence find $f^{-1}$.

16. (i) Let $R$ be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = [(a, b) : 2 \text{ divides } (a - b)]$. Write the equivalence class $[0]$.

(ii) Let $R$ be a relation on set of natural number $N$ as follows. $R = \{(x, y) : x, y \in N \text{ and } 2x + y = 24\}$. Find the domain and range of the relation $R$. Is $R$ an equivalence relation or not?

17. A binary operation $*$ is defined on the set $X = R - \{-1\}$ by $x * y = x + y + xy, \forall x, y \in X$. Check whether $*$ is commutative and associiative. Find the identity element and also the inverse of each element of $X$.

18. Let $N$ denote the set of all natural numbers and $R$ be the relation on $N \times N$ defined by $(a, b) R (c, d)$ if $ad(b + c) = bc(a + d)$. Show that $R$ is an equivalence relation.

19. Consider $f : R_+ \to (-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that $f$ is invertible with $f^{-1}(y) = \frac{\sqrt{54 + 5y} - 3}{5}$.

20. Let $f : [-1, \infty] \to [-1, \infty]$ is given by $f(x) = (x + 1)^2 - 1$. Show that $f$ is invertible.

Also, find the set $S = \{x : f(x) = f^{-1}(x)\}$. 

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SmartSkills Sanskriti School

Class XII / Mathematics/7
Assignment No. 2
Inverse Trigonometric Functions

Q 1 – 7 are very short answer type questions

Q1. For the principal values, evaluate \( \tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) \).

Q2. Find the principal value of \( \sec^{-1}\left(-\frac{1}{2}\right) \).

Q3. Evaluate: \( \tan\left(2\tan^{-1}\frac{1}{2}\right) \).

Q4. If \( 4\sin^{-1}x + \cos^{-1}x = \pi \), find the value of \( x \).

Q5. In triangle ABC, if C is a right angle, then evaluate \( \tan\left(\frac{a}{b+c}\right) + \tan\left(\frac{b}{c+a}\right) \).

Q6. If \( \sin^{-1}x + \cos^{-1}x = \frac{\pi}{6} \), find \( x \).

Q7. If \( \sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1}x \), prove that \( x = \frac{a+b}{1-ab} \).

Q8. Solve for \( x \): \( 6\sin^{-1}x + \sin^{-1}6\sqrt{3}x = \frac{-\pi}{2} \).

Q9. If \( (\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8} \), then find \( x \).

Q10. Prove that \( \cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right) = \frac{\pi}{3} \).

Q11. Simplify each of the following:

a. \( \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right), x \neq 0 \)

b. \( \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) \)

c. \( \sin^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right) \)

Q12. Prove that: \( \tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right) = \begin{cases} \frac{\pi}{4} + \frac{x}{2}, & 0 < x < \frac{\pi}{2} \\ \frac{\pi}{4} - \frac{x}{2}, & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases} \)

Q13. Prove: \( \sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2} \).

Q14. Solve for \( x \): \( \tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31} \).

Q15. Solve for \( x \): \( \sin^{-1}(1-x) + \sin^{-1}x = \cos^{-1}x \).
Assignment No. 3

MATRICES

Questions 1-5 are very short answer type questions

1. Find the values of \(x, y\), if
\[
\begin{bmatrix}
1 & 3 \\
0 & x
\end{bmatrix} +
\begin{bmatrix}
y & 0 \\
1 & 2
\end{bmatrix} =
\begin{bmatrix}
5 & 6 \\
1 & 8
\end{bmatrix}
\]

2. If \(A\) is a square matrix, such that \(A^2 = A\), then find \((I + A)^2 - 3A\)

3. Construct a \(3 \times 2\) matrix \(A\), whose elements are given by \(a_{ij} = \frac{1}{2}|2i - j|\)

4. For a skew symmetric matrix \(A = [a_{ij}]\), what is the nature of element \(a_{ij}\), if \(i = j\)

5. Find “\(X\)” if
\[
\begin{bmatrix}
2 & -1 \\
3 & -1
\end{bmatrix} +
\begin{bmatrix}
2 & 4 \\
5 & 0
\end{bmatrix}
\]

6. If \(A =
\begin{bmatrix}
-1 \\
2 \\
3
\end{bmatrix}, B =
\begin{bmatrix}
-2 & -1 & -4
\end{bmatrix}\), verify that \((AB)^T = B^T A^T\)

7. Express
\[
\begin{bmatrix}
1 & 4 & 3 \\
-2 & 5 & 8
\end{bmatrix}
\]
as the sum of symmetric and skew symmetric matrix.

8. Find the value of “\(x\)” which satisfy
\[
\begin{bmatrix}
x & 4 & 1 \\
0 & 2 & 4 \\
0 & 2 & -4
\end{bmatrix}
\]

9. If \(A =
\begin{bmatrix}
\alpha & 1 \\
0 & \alpha
\end{bmatrix}\), then prove by the principle of mathematical induction that
\(A^n =
\begin{bmatrix}
\alpha^n & n\alpha^{n-1} \\
0 & \alpha^n
\end{bmatrix}\).

10. If \(A =
\begin{bmatrix}
1 & 2 & 0 \\
3 & -4 & 5 \\
0 & -1 & 3
\end{bmatrix}\), and \(f(x) = x^2 - 4x + 3\), then find \(f(A)\)
Questions 1-6 are very short answer type questions

1. If for a matrix $A$, $|A| = 4$, find $3|A|$, where matrix $A$ is of order $2 \times 2$

2. $A$ is a non singular matrix of order $3$ and $|A| = -5$, find $|adj A|$.

3. If $\begin{bmatrix} a_{ij} \end{bmatrix}$ is a matrix of order $3 \times 3$, find the value of $a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$

4. If $A$ is a square matrix of order $3 \times 3$, and $|4A| = k|A|$, find $k$

5. If $\begin{bmatrix} a_{ij} \end{bmatrix}$ is a matrix of order $2 \times 2$, such that $|A| = -15$, then find $a_{11}C_{21} + a_{22}C_{22}$

6. If $A$ is a square matrix of order $3 \times 3$ find $|A adj A|$ if $|A| = 6$.

7. Using matrix method solve the following system of linear equations:
   a) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$
   b) $5x + 3y + z = 16, 2x + y + 3z = 19, x + 2y + 4z = 25$

8. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find $AB$ and hence solve $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$

9. If $A = \begin{bmatrix} 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find $A^{-1}$, and hence solve $2x - y + 3z = 4, x + 2y - 3z = 0$

10. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$, then verify that $A^2 - 12A - I = 0$, where $I$ is a unit matrix of order $2$ and hence find $A^{-1}$.

11. Find the matrix “$A$” for which $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. Area of the triangle with vertices $(-2, 4), (2, k), (5, 4)$ is 35 square units. Find “$k$".
13. Using properties of determinants prove that
\[
\begin{vmatrix}
(b+c)^2 & a^2 & a^2 \\
b^2 & (c+a)^2 & b^2 \\
c^2 & c^2 & (a+b)^2
\end{vmatrix} = 2abc(a+b+c)^3.
\]

14. Using properties of determinants prove that
\[
\begin{vmatrix}
a & a+b & a+b+c \\
2a & 3a+2b & 4a+3b+2c \\
3a & 6a+3b & 10a+6b+3c
\end{vmatrix} = a^3.
\]

15. Using properties of determinants prove that
\[
\begin{vmatrix}
b^2 + c^2 & ab & ac \\
ab & c^2 + a^2 & bc \\
ca & bc & a^2 + b^2
\end{vmatrix} = 4a^2b^2c^2.
\]
Assignment No. 5

Continuity and Differentiability

Questions 1 – 10 are very short answer type questions

1. If \( f(1) = 4 \), \( f'(1) = 2 \), find the value of derivative of \( \log f(e^x) \) w.r.t. “x” at \( x = 0 \)

2. Find \( \frac{dy}{dx} \), if \( x^6 + y^6 = 30 \)

3. If \( y = e^{x^2} + x^2 - x + 1 \), Prove that \( \frac{dy}{dx} = \frac{y}{1-y} \).

4. Differentiate \( \tan^{-1}\left( \frac{2x}{1-x^2} \right) \) with respect to “x”

5. State the points of discontinuity for the function \( f(x) = [x] \) in \(-3 < x < 3\)

6. Differentiate \( \tan^{-1}\left( \frac{\sqrt{1+x^2} - 1}{x} \right) \) with respect to “x”.

7. If \( y = \tan^{-1}\left( \frac{5x}{1-6x^2} \right) \), Prove that \( \frac{dy}{dx} = \frac{2}{1+4x^2} + \frac{3}{1+9x^2} \).

8. Differentiate \( \tan^{-1}\left( \frac{1-\cos x}{1+\cos x} \right) \) with respect to \( \tan^{-1} x \).

9. If “f” is a differentiable function at “x”=1 such that \( f(1) = 5 \), \( f'(1) = \frac{1}{5} \) & \( g = f^{-1} \), then find \( g'(5) \)

10. Differentiate w.r.t. x: \( a^{\sin^{-1} x} \)

11. For what value of “k” the function \( f(x) = \begin{cases} \frac{\sqrt{5x+2} - \sqrt{4x+4}}{x-2} &; x \neq 2 \\ k &; x = 2 \end{cases} \) is continuous at \( x = 2 \).

12. Determine the value of “a”, “b”, and “c” if the following function is continuous at

\[
x = 0 \quad f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} &; \text{when } x > 0 \\ c &; \text{when } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} &; \text{when } x < 0 \end{cases}
\]
13. Discuss the derivability of the function  
\[ f(x) = \begin{cases} 
    x-1, & x < 2 \\
    2x-3, & x \geq 2 
\end{cases} \text{ at } x = 2 
\]

14. Find “a” and “b” , if the function given by  
\[ f(x) = \begin{cases} 
    x^2, & x \leq 1 \\
    ax+b, & x > 1 
\end{cases} \] 

15. Discuss the continuity of the function  
\[ f(x) = |x-1| - |x-2|. \]

16. Verify Rolle’s theorem for the function  
\[ f(x) = e^x (\sin x - \cos x) \] 
on \( \left[ \frac{\pi}{4}, \frac{5\pi}{4} \right] \)

17. If  
\[ y = x^{\sin x} + \sin^{-1}\sqrt{x}, \] 
find \( \frac{dy}{dx} \).

18. If  
\[ x^p \cdot y^q = (x+y)^{p+q}, \] 
prove that  
\[ \frac{dy}{dx} = \frac{y}{x}. \]

19. If  
\[ y = \sin(m \sin^{-1} x), \] 
prove that  
\[ (1-x^2)y_2 - xy_1 + m^2 y = 0. \]

20. If  
\[ x = \sin t, \ y = \sin pt, \] 
prove that  
\[ (1-x^2)y_2 - xy_1 + p^2 y = 0. \]

21. If  
\[ \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y), \] 
show that  
\[ \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}. \]
Assignment No. 6
Application of Derivatives

1. Water is dripping out from a conical funnel of semi vertical angle \( \frac{\pi}{4} \) at a uniform speed of \( 2 \text{ cm}^3 / \text{sec} \) through a tiny hole at the vertex of the bottom. When the slant height of water is 4 cm, find the rate of decrease of slant height of the water.

2. A man is moving away from a tower 49.6 m high at the rate of 2 m/s. Find the rate at which the angle of elevation of the top of the tower is changing, when he is at a distance of 36 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.

3. Evaluate following up to three decimal places using differentiation:
\[ \sqrt{25.2}, \sqrt{29}, \sqrt{0.037} \]

4. Find the intervals in which the function \( f(x) = \log(1+x) - \frac{2x}{2+x} \) increasing or decreasing.

5. Find the intervals in which the function \( f(x) = (x+1)^3(x-3)^3 \) is increasing or decreasing. Also find the points at which the function has local maxima, local minima and the point of inflexion.

6. Find all the points of local maximum and minimum and the corresponding maximum and minimum values of the following function \( \frac{3}{4}x^4 - 8x^3 + \frac{45}{2}x^2 + 105 \).

7. Find the point on the curve \( y^2 = 4x \) which is nearest to the point \( (2,-8) \)

8. Find the equation of the tangent to the curve \( y = (x^3-1)(x-2) \) at the points where the curve cuts the x-axis.

9. Find the intervals in which the function \( f(x) = 2x^3 - 9x^2 + 12x + 15 \) is increasing and decreasing.

10. Separate \( \left[ 0, \frac{\pi}{2} \right] \) into sub intervals in which \( f(x) = \sin^4 x + \cos^4 x \) is increasing or decreasing.
11. Find the points of local maxima and local minima and also the local maximum and local minimum values of the following functions: 

(i) \( f(x) = 2 \cos x + x, x \in (0, \pi) \)

(ii) \( f(x) = 2 \sin x - x, x \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \)

12. Find the equation of the tangent and normal to the curve 

\[ x = 1 - \cos \theta; \quad y = \theta - \sin \theta \] at \( \theta = \frac{\pi}{4} \)

13. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of cone.

14. An open box with a square base is to be made of given iron sheet of area 27 sq.m. Show that the maximum volume of the box is 13.5 cu.m.

15. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
Assignment No. 7(a)

Indefinite Integrals-1

(Method of Substitution, Trigonometric Identities, Special Integrals)

Q1-10 are very short answer type questions

1. Evaluate: \( \frac{x - 2}{\sqrt{x^2 + 1}} \)

2. Evaluate: \( \int x^{\frac{3}{2}} (1 + x^{\frac{3}{2}}) \, dx \)

3. Write a value of \( \int \frac{1}{\sqrt{3 \sin x + \cos x}} \, dx \)

4. Write a value of \( \int \frac{1 - \tan x}{x + \log \cos x} \, dx \)

5. If \( f'(x) = \frac{4}{x^2} \) and \( f(1) = 6 \), find \( f(2) \)

6. Write a value of \( \int (\cos(\log x) + \sin(\log x)) \, dx \)

7. If \( \int \frac{2^x}{\sqrt{1 - 4^x}} \, dx = k \sin^{-1}(2^x) + c \), then what is the value of \( k \)?

8. Find a value of \( \int \frac{1}{9x^2 - 16} \, dx \)

9. Evaluate \( \int \frac{x}{e^{x^2}} \, dx \)

10. Evaluate: \( \int \frac{dx}{e^x + e^{-x}} \)

11. Evaluate: \( \int \frac{1}{(\sin x - 2 \cos x)(2 \sin x + \cos x)} \, dx \)

12. Evaluate: \( \int \frac{1}{\sqrt{\sin^3 x(\sin x + 2 \cos x)}} \, dx \)

13. Evaluate: \( \int \csc^8 x \, dx \)

14. Evaluate: \( \int 5^{\frac{5}{x}} \cdot 5^{\frac{x}{5}} \cdot 5^x \, dx \)

15. Evaluate: \( \int \frac{x^2}{\sqrt{x^6 - a^6}} \, dx \)

16. Evaluate: \( \int \frac{x^3 + x^2 + 2x + 1}{x^2 - x + 1} \, dx \)

17. Evaluate: \( \int \frac{3-x}{x-2} \, dx \)

18. Evaluate: \( \int \frac{e^x}{2e^{2x} + 3e^x + 1} \, dx \)

19. Evaluate: \( \int \frac{6x + 5}{\sqrt{6 + x - 2x^2}} \, dx \)

20. Evaluate: \( \int \sqrt{1 + \csc x} \, dx \)
21. \( \frac{1}{(3\sin x + \cos x)^2} \)

22. \( \frac{1}{x\log x \log(\log x)} \)

23. \( \frac{1}{\sin^2 x + \sin 2x} \)

24. \( \sqrt{\tan \theta} + \sqrt{\cot \theta} \)

25. \( \frac{1}{\sec x - 1} \)

26. \( \frac{\log x^2}{x} \)

27. \( \frac{x^2 + 9}{x^4 + 81} \)

28. \( \frac{1}{5 + 7\cos x + \sin x} \)
Assignment No. 7(b)

Indefinite Integrals-2

(By Parts, Partial Fraction& three more special integrals)

Q: Integrate following functions with respect to x:

1. \( \frac{x^2}{(x-1)(x+1)^2} \)

2. \( \cos^{-1}(4x^3 - 3x) \)

3. \( e^x \left[ \frac{x-1}{(x+1)^3} \right] \)

4. \( \frac{\tan x + \tan^3 x}{1 + \tan^3 x} \)

5. \( \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-\frac{x}{2}} \)

6. \( \frac{x \tan^{-1} x}{(1 + x^2)^{\frac{3}{2}}} \)

7. \( \frac{\log(x+2)}{(x+2)^2} \)

8. \( \frac{1}{\sin x(5 - 4 \cos x)} \)

9. \( e^{\sqrt{x}} \)

10. \( \cos^3 \sqrt{x} \)

11. \( e^{-3x} \cos 2x \)

12. \( e^{x}(\tan x - \log \cos x) \)

13. \( \frac{e^{2x} \sin 4x - 2}{1 - \cos 4x} \)

14. \( \frac{x^3 - 1}{x^3 + x} \)

15. \( (x+1)\sqrt{3-x-x^2} \)

16. \( x^2 \cot ec^{-1}x \)

17. \( \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] \)
Integrate following functions w.r.t. $x$

1. $\frac{x}{x^4 + x^2 + 1}$
2. $\frac{\sin x}{\sqrt{\cos^2 x - 2 \cos x - 3}}$
3. $\frac{\sin^3 x}{\sqrt{\cos x}}$
4. $\frac{1}{1 + \sqrt{x}}$
5. $\frac{1}{x \log x \log(\log x)}$
6. $\frac{1}{x(x^2 + 1)}$
7. $\frac{1}{\sqrt{x} + \sqrt{x}}$
8. $\frac{1}{(3 \sin x + \cos x)^2}$
9. $\frac{1}{\sin x + \sqrt{3} \cos x}$
10. $\cos^7 x$
11. $\frac{1 - x^2}{x(1 - 2x)}$
12. $\frac{\tan x}{\sqrt{\sin^4 x + \cos^4 x}}$
13. $\frac{\cos 2x}{\sin x}$
14. $\frac{\sin x + \cos x}{\sqrt{\sin 2x}}$
15. $\frac{1}{\sqrt{2x - x^2}}$
16. $\frac{x^2 + 5x + 3}{x^2 + 3x + 2}$
17. $\frac{2 \sin 2x - \cos x}{6 - \cos^2 x - 4 \sin x}$
18. $\frac{x^2 + 9}{x^4 + 81}$
19. $\frac{1}{4 \sin^2 x + 5 \cos^2 x}$
20. $\frac{1}{3 \sin^2 x + 8 \cos^2 x + 1}$
21. $\frac{1}{2 - 3 \cos 2x}$
22. $\frac{\cos x}{\cos 3x}$
23. $\frac{1}{\sec x + \cos ecx}$
24. $\frac{1}{(e^x + e^{-x})^2}$
25. $\frac{\sin 2x}{\sin^4 x + \cos^4 x}$
26. $\frac{1}{1 + \cot x}$
27. $\sqrt{\cot \theta}$
28. \( \frac{ax^3 + bx}{x^4 + c^2} \)  
29. \( \sqrt{\frac{1+x}{x}} \)  
30. \( (\sin^{-1} x)^3 \)  

31. \( \frac{x^2 + x + 1}{(x+1)^2(x+2)} \)  
32. \( \frac{1}{\sin x(2 + 3 \cos x)} \)  
33. \( \frac{\cos x}{1 + \cos x} \)  

34. \( \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] \)  
35. \( e^x \left( 1 - x \right)^2 \)  
36. \( \frac{1}{1 + x + x^2 + x^3} \)  

37. \( \tan^{-1} \frac{1-x}{\sqrt{1+x}} \)  
38. \( \frac{1}{a + b \tan x} \)  
39. \( \sqrt{1 + x - 2x^2} \)  

40. \( \frac{\log(1-x)}{x^2} \)  
41. \( e^x \left[ \frac{1 - \sin x}{1 - \cos x} \right] \)  
42. \( e^x \left[ \frac{x - 1}{(x+1)^3} \right] \)  

43. \( (1-2x)\sqrt{4-3x-3x^2} \)  
44. \( \sqrt{\tan \theta} + \sqrt{\cot \theta} \)  
45. \( \left[ \sqrt{1-\sin x} \right] \left[ \frac{1}{1 + \cos x} \right] e^{-\theta/2} \)  

46. \( \frac{x^3 - 1}{x^3 + x} \)  
47. \( (\log x)^2 \)  
48. \( \sin x \sin 2x \sin 3x \)  

49. \( \frac{1}{\sin x(5 - 4 \cos x)} \)  
50. \( \frac{1}{(x-1)^2(x^2 + 2)} \)  
51. \( \frac{\sin x}{(2 + \cos x)(1 - 3 \cos x)} \)
Q1 - 8 are very short answer type questions.

1. Evaluate, \( \int_{0}^{1.5} [x] \, dx \) (where \([x]\) is greatest integer function)

2. If \( \int_{0}^{1} (3x^2 + 2x + k) \, dx = 0 \), then find "k"

3. If \( \int_{0}^{a} 3x^2 \, dx = 8 \), then find the value of “a”

4. Evaluate \( \int_{-1}^{1} |1 - x| \, dx \)

5. Evaluate \( \int_{0}^{1} e^{11} \, dx \)

6. If \( \int_{0}^{k} \frac{dx}{2 + 8x^2} = \frac{\pi}{16} \), find the value of k.

7. Evaluate \( \int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{1 + \sqrt{\cot x}} \, dx \)

8. If \( \int_{a}^{b} x^3 \, dx = 0 \) and \( \int_{a}^{b} x^2 \, dx = \frac{2}{3} \), find the value of a and b.

9. Evaluate the following definite integrals:

   a) \( \int_{0}^{\frac{\pi}{4}} 2 \tan^3 x \, dx \)
   b) \( \int_{0}^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) \, dx \)
   c) \( \int_{0}^{\pi} \frac{dx}{6 - \cos x} \)
   d) \( \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{dx}{1 + \cot^{\frac{3}{2}} x} \)
   e) \( \int_{0}^{\pi} e^{\cos x} \cos x \, dx \)
   f) \( \int_{0}^{\pi} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx \)
   g) \( \int_{0}^{1} \cot^{-1} (1 - x + x^2) \, dx \)
   h) \( \int_{0}^{1} \log(1 + x) \, dx \)
   i) \( \int_{1}^{3} |x^2 - 2x| \, dx \)
   j) \( \int_{0}^{\pi} e^{\cos x} + e^{-\cos x} \, dx \)
10. Evaluate following definite integrals as limit of a sum:

i. \( \int_{0}^{1} (x^2 - 3x) \, dx \)  

ii. \( \int_{1}^{3} e^{2x} \, dx \)  

iii. \( \int_{-2}^{2} (3x^2 - 2x + 4) \, dx \)  

iv. \( \int_{0}^{4} (x + e^x) \, dx \)
Assignment No. 9

Applications of Integrals - Area of the bounded regions

1. Find the area of the region bounded by the curve \( y^2 = x - 2, x = 4, x = 6 \) and the x axis in the first quadrant using integration.

2. Sketch the region common to the circle \( x^2+y^2=16 \) and the parabola \( x^2=6y \). Also find the area of the region using integration.

3. Find the area of the region bounded between the parabolas \( y^2 = 4ax, x^2 = 4ay \), where \( a>0 \).

4. Using Integration find the area of the triangle ABC whose vertices has coordinates given by \( A(2, 5), B(4, 7), C(6, 2) \)

5. Compute the area bounded by the lines \( x + 2y = 2, \ y - x = 1 \ & \ 2x + y = 7 \)

6. Find the area of the region bounded by the curve \( y^2 = 2y - x \) and the y axis.

7. Using integration find the area of the region \( \{ (x,y) : |x| \leq y \leq \sqrt{4-x^2} \} \)

8. Find the area bounded by the triangle whose vertices are \( (0,0), (2,4),(4,-2) \).

9. Sketch the graph of \( f(x) = \begin{cases} \frac{|x-2|+2}{x}, & x \leq 2 \\ \frac{2}{x^2-2}, & x > 2 \end{cases} \). Evaluate \( \int_0^4 f(x)dx \), what does the value of this integral represent on the graph?

10. Find the area bounded by the curves \( y = 6x - x^2 \) and \( y = x^2 - 2x \)

11. Find the area of the region \( \{ (x,y) : x^2 + y^2 \leq 1 \leq x + y \} \)

12. Find the area of the region enclosed between the circles \( x^2 + y^2 = 4, (x-2)^2 + y^2 = 1 \).

13. Using integration find the area of the region \( \{ (x,y) : |x-1| \leq y \leq \sqrt{5-x^2} \} \)
Assignment No. 10
Differential Equations

Q1-4 and each part of Q5&6 is a very short answer type question.

1. Find the differential equation corresponding to \( y = Ae^x + Be^{-x} \)

2. Find a solution of the differential equation \( \frac{dy}{dx} = \frac{y}{x} \).

3. Show that the differential equation \( x\frac{dy}{dx} - y = \sqrt{x^2 + y^2} \) is homogenous.

4. Determine the order and degree (if defined) of each of the following differential equations:
   
i. \( \left( \frac{d^2x}{dt^2} \right)^4 - 7t \left( \frac{dx}{dt} \right)^3 = \log t \)
   
   ii. \( \left( \frac{d^2y}{dx^2} \right)^2 + \sin \left( \frac{dy}{dx} \right) = 0 \)
   
   iii. \( 1 + \left( \frac{dy}{dx} \right)^2 = 2x - \frac{dy}{dx} \)
   
   iv. \( \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^3 = 5 \left( \frac{d^2y}{dx^2} \right)^3 \)

5. Write the integrating factor of the following differential equations:
   
i. \( (1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x \)
   
   ii. \( x\frac{dy}{dx} - y = \log x \)
   
   iii. \( \frac{dx}{dy} - \frac{2x}{y} = 3y^3 - 5y + 1 \)
   
   iv. \( \frac{dy}{dx} + 2y = xe^{4x} \)

6. Solve the following differential equations:
   
i. \( (x-1)\frac{dy}{dx} = 2xy, \) given that \( x = 2, y = 1 \)
   
   ii. \( (1 + x)yd x + (1 + y)xd y = 0 \)
   
   iii. \( \frac{dy}{dx} + 2y = e^{-2x} \sin x, \) given \( x = 0, y = 0 \)
   
   iv. \( yd x - (x + 2y^2)d y = 0, \) given that \( x = 2 \) when \( y = 1. \)
   
   v. \( \frac{dy}{dx} = \cos (x + y) + \sin (x + y) \)
   
   vi. \( \sqrt{1 + x^2 + y^2 + x^2y^2 + xy} \frac{dy}{dx} = 0 \)
   
   vii. \( (x + y + 1)\frac{dy}{dx} = 1 \)
   
   viii. \( (1 + e^{2x})\frac{dy}{dx} + (1 + y^2)e^x dx = 0, \) given that \( x = 0, y = 1 \)
   
   ix. \( \frac{dy}{dx} = 1 - x + y - xy \)
Assignment No.11 (a)

VECTORS

Very short answer type questions

1. Write down a unit vector in XY plane making an angle of 60° with the positive direction of x-axis.

2. Find the distance of the point \((a, b, c)\) from the z-axis.

3. Give an example of two vectors \(\vec{a}\) and \(\vec{b}\) such that \(|\vec{a}| = |\vec{b}|\) but \(\vec{a} \neq \vec{b}\).

4. If \(\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}\) and \(\vec{b} = 6\hat{i} + \lambda\hat{j} + 3\hat{k}\), such that they are collinear vectors, then find \(\lambda\).

5. Write the direction cosines of the vector \(3\hat{i} - 6\hat{j} + 2\hat{k}\).

6. If the magnitude of the position vector of the point \((3, -2, p)\) is 7 units, find all possible values of \(p\).

7. A vector is inclined at \(\frac{\pi}{4}\), \(\frac{\pi}{3}\) with the “x” and “y” axes respectively. Find the angle it makes with the “z” axis.

8. If \(\vec{a} = p\hat{i} + 3\hat{j}\) and \(\vec{b} = 4\hat{i} + p\hat{j}\), find the values of \(p\) so that \(\vec{a}\) and \(\vec{b}\) may be parallel.

9. Write the position vector of the point dividing the line segments joining the points with position vectors \(\vec{a}\) and \(\vec{b}\) in the ratio 1:4 externally, where \(\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}\) and \(\vec{b} = -\hat{i} + \hat{j} + \hat{k}\).

10. Find the angle at which the following vectors are inclined to each of the coordinate axes:

    (i) \(\hat{i} + \hat{j} - \hat{k}\)  
    (ii) \(-\hat{i} - \hat{j}\)

11. Show that \(\cos\alpha \cos\beta \hat{i} + \cos\alpha \sin\beta \hat{j} + \sin\alpha \hat{k}\) is a unit vector.

12. Find a unit vector perpendicular to the vectors \(\hat{i} - 3\hat{j} + \hat{k}\) and \(2\hat{i} - \hat{j} + \hat{k}\).

13. Find a vector of magnitude 3 units, which is orthogonal to the vectors \(3\hat{i} + \hat{j} - 4\hat{k}\) and \(6\hat{i} + 5\hat{j} - 2\hat{k}\).

14. If \(\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}\) \& \(\vec{b} = -\hat{i} + 3\hat{j} - 2\hat{k}\), then find \(|\vec{a} - 2\vec{b}|\).

15. Find the scalar and vector projection of \(2\hat{i} - \hat{j} + \hat{k}\) on \(\hat{i} - 2\hat{j} + \hat{k}\).

16. Show that the three points A \((3, -5, 1)\) B \((-1, 0, 8)\) and C \((7, -10, -6)\) are collinear.
17. For what value of ‘p’, the vectors \( \vec{a} = 3\hat{i} - \hat{j} + 4\hat{k} \) and \( \vec{b} = p\hat{i} + 3\hat{j} + 3\hat{k} \) are perpendicular to each other?

18. If \( \vec{a} = x\hat{i} + 3\hat{j} - 6\hat{k} \) and \( \vec{b} = 2\hat{i} - \hat{j} + y\hat{k} \), find the value of x and y so that \( \vec{a} \) and \( \vec{b} \) may be collinear.

19. Find the position vector of the midpoint of the line segment joining the points \( A(5\hat{i} + 3\hat{j}) \) and \( B(3\hat{i} - \hat{j}) \)

20. If \( |\vec{a}| = 2, |\vec{b}| = 5 \) and \( |\vec{a} \times \vec{b}| = 8 \), find \( \vec{a}.\vec{b} \).

21. A line makes angles of 45° and 45° with x and y axis respectively. Find the angle it makes with z-axis.

22. Evaluate: \( \hat{i}.(\hat{j} \times \hat{k}) + j.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{j} \times \hat{i}) \)

**Assignment No. 11(b)**

**VECTORS**

1. The vectors \( \vec{a} \) and \( \vec{b} \) are non-zero non-collinear and \( \vec{c} = (p + 4q)\vec{a} + (2p + q + 1)\vec{b} \), and \( \vec{d} = (-2p + q + 2)\vec{a} + (2p - 3q - 1)\vec{b} \). Then find the values of p and q so that \( 3\vec{c} = 2\vec{d} \).

2. If \( \vec{a} \) and \( \vec{b} \) are unit vectors, then what is the angle between \( \vec{a} \) and \( \vec{b} \) so that \( \vec{a} - \sqrt{2} \vec{b} \) is a unit vector?

3. If \( |\vec{a}| = 3, |\vec{b}| = 5 \) and \( \vec{a}.\vec{b} = -8 \), find \( |\vec{a} + \vec{b}| \).

4. The adjacent sides of a parallelogram are represented by the vectors \( \vec{a} = \hat{i} + \hat{j} - \hat{k} \) and \( \vec{b} = -2\hat{i} + \hat{j} + 2\hat{k} \). Find unit vectors parallel to the diagonals of the parallelogram.

5. Compute area of a parallelogram whose diagonals are the vectors \( 2\hat{i} - 3\hat{j} + 6\hat{k} \) and \( 2\hat{i} - 2\hat{j} - \hat{k} \).

6. If \( A, B, C \) have position vectors \( (2, 0, 0), (0, 1, 0), (0, 0, 2) \), show using vectors that triangle ABC is isosceles.

7. Determine \( \lambda \) and \( \mu \) if \( (2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0} \)

8. Dot product of a vector with vectors \( \hat{i} + \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}, \) and \( \hat{i} + 3\hat{j} + 4\hat{k} \) is respectively 7, 16 and 22. Find the vector.
9. If \( \vec{a}, \vec{b}, \vec{c} \) are unit vectors s.t. \( \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0 \) and angle between \( \vec{b} \) and \( \vec{c} \) is \( \frac{\pi}{6} \), prove that 
\[
\vec{a} = \pm 2(\vec{b} \times \vec{c}).
\]

10. Find the volume of the parallelepiped whose sides are given by \(-3\hat{i} + 7\hat{j} + 5\hat{k}, -5\hat{i} + 7\hat{j} - 3\hat{k} \& 7\hat{i} - 5\hat{j} - 3\hat{k}\)

11. Show that the vectors \(-2\hat{i} - 2\hat{j} + 4\hat{k}, -2\hat{i} + 4\hat{j} - 2\hat{k} \& 4\hat{i} - 2\hat{j} - 2\hat{k}\) are coplanar.

12. Find \( \lambda \), for which the four points with position vectors \(-\hat{j} - \hat{k}, 4\hat{i} + 5\hat{j} + \lambda\hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k} \& -4\hat{i} + 4\hat{j} + 4\hat{k}\) are coplanar.

13. Prove that 
\[
\{\vec{a} - \vec{b}\} \cdot \{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\} = 0
\]

14. Evaluate: \[
\left[ 2\hat{i} \hat{j} \hat{k} \right] + \left[ \hat{i} \hat{k} \hat{j} \right] + \left[ \hat{k} \hat{j} 2\hat{i} \right]
\]

15. For any three vectors \( \vec{a}, \vec{b}, \vec{c} \), prove that 
\[
\left[ \vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a} \right] = 2\left[ \vec{a}, \vec{b}, \vec{c} \right]
\]

16. Determine \( \alpha \) such that a vector \( \vec{r} \), is at right angles to each of the three vectors 
\[
\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k} \& \vec{c} = -2\hat{i} + \hat{j} + 3\hat{k}
\]

17. If \( \vec{a}, \vec{b}, \vec{c} \) are three vectors s.t. \( \vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a} \). Prove that \( \vec{a}, \vec{b}, \vec{c} \) are mutually at right angles and \( |\vec{b}| = 1, |\vec{a}| = |\vec{c}|. \)

18. \( \vec{a} \times \vec{b} = \vec{c} \times \vec{d}, \vec{a} \times \vec{c} = \vec{b} \times \vec{d}, \) Prove that \( \vec{a} - \vec{d} \) is parallel to \( \vec{b} - \vec{c}, \) provided \( \vec{a} \neq \vec{d} \) and \( \vec{b} \neq \vec{c}. \)

19. Find two vectors of unit length which make angles of 45° with the position vector of \((1, 0, 0)\) and are at right angles to the position vector of \((0, 0, 1)\).

20. Find vector \( \vec{c} \) such that \( \vec{c} \hat{i} = \vec{c} \hat{j} = \vec{c} \hat{k} \) and \( |\vec{c}| = 100. \)
Three Dimensional Coordinate Geometry

Q1-10 are very short answer type questions.

1. Find the perpendicular distance of the plane $\vec{r}.(5\hat{i} - 3\hat{j} + 4\hat{k}) + 9 = 0$ from origin.

2. Find vector equation of the plane which is at a distance of 3 units from origin and has $\hat{j}$ as the unit normal.

3. Write the equation of the plane passing through the point $(2, -1, 1)$ and parallel to the plane $3x + 2y - z = 7$.

4. If the lines $\frac{x-1}{-3} = \frac{y-2}{2p} = \frac{z-3}{2}$ and $\frac{x-1}{3p} = \frac{y-1}{-1} = \frac{z-6}{-5}$ are perpendicular to each other, then find the value of $p$.

5. Find the equation of the line passing through the point $2\hat{i} - 3\hat{j} + 4\hat{k}$ parallel to the line $\vec{r} = \hat{i} - 3\hat{j} - 5\hat{k} + \lambda(2\hat{i} + 5\hat{k})$.

6. Write position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ & $\vec{b}$ externally in the ratio 1:4, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$.

7. The cartesian equations of a line are $3x + 1 = 6y - 2 = 1 - z$. Find the fixed point through which it passes and its direction ratios. Also find the vector equation.

8. Find the angle between $\frac{x-1}{2} = \frac{2-y}{-1} = \frac{-z-3}{2}$ and $x + y + 4 = 0$.

9. Find the intercepts cut off by the plane $3x + 2y + z = 7$.

10. Write the equation of the plane parallel to XOY plane passing through the point $(1, -2, 5)$.

11. Find the foot of the perpendicular from P $(1, 2, 3)$ on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also obtain the equation of the plane containing the line and the point $(1, 2, 3)$.

12. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) = 3$ and $\vec{r}.(5\hat{i} - 3\hat{j} + 4\hat{k}) + 9 = 0$ and parallel to the line $\vec{r} = \hat{i} + 3\hat{j} + 5\hat{k} + \lambda(2\hat{i} + 4\hat{j} + 5\hat{k})$. 
13. Find the equation of the plane through the points \((1, 0, -1)\), \((3, 2, 2)\) and parallel to the line \(\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{-3}\).

14. Find the shortest distance between the lines:
   a) \(\vec{r} = (-4\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})\)
   b) \(\vec{r} = (3 + \lambda\hat{i} + (5 - 2\lambda)\hat{j} + (7 + \lambda)\hat{k})\)
   \(\vec{r} = (7\mu - 1)\hat{i} + (-1 - 6\mu)\hat{j} + (\mu - 1)\hat{k}\)

15. Find the equation of the plane through the points \((2,1,-1)\), \((-1, 3, 4)\) and perpendicular to the plane \(x - 2y + 4z = 10\).

16. Show that the four points \((0, -1, -1)\), \((4, 5, 1)\), \((3, 9, 4)\) and \((-4, 4, 4)\) are coplanar. Also find the equation of the plane containing them.

17. Find the equation of the plane passing through the line of intersection of the planes \(\vec{r}.(\hat{i} + 3\hat{j}) + 6 = 0\) and \(\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0\), which is at a unit distance from the origin.

18. Find the equation of the plane passing through the line of intersection of the planes \(4x - y + z = 10\) and \(x + y - z = 4\) and parallel to the line with direction ratios proportional to 2, 1, 1. Find also the perpendicular distance of \((1, 1, 1)\) from this plane.

19. Show that the lines \(\vec{r} = 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})\) and \(\vec{r} = 2\hat{i} + 6\hat{j} + 3\hat{k} + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})\) are coplanar. Also find the equation of the plane containing them.

20. Prove that the image of the point \((3,-2,1)\) in the plane \(3x - y + 4z = 2\) lies on the plane \(x + y + z + 4 = 0\).

21. Find the distance of the point \((3,4,5)\) from the plane \(x + y + z = 2\) measured parallel to the line \(2x = y = z\).

22. Show that the line \(\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})\) is parallel to the plane \(\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5\). Also, find the distance between them.

23. Find the coordinates of the point where the line through \((5, 1, 6)\) and \((3, 4, 1)\) crosses the \(ZX\) plane.

24. Find the equations of the planes parallel to the plane \(x - 2y + 2z - 3 = 0\) and which are at a unit distance from the point \((1,1,1)\).
Assignment No. 13

Probability

(Q1-Q5 are very short answer questions)

1. If \( P(A) = \frac{1}{4}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{8} \), then find \( P(\text{not } A \text{ and not } B) \).

2. If \( P(A \cup B) = 0.8, \ P(A \cap B) = 0.3 \), then find \( P(\overline{A}) + P(\overline{B}) \).

3. A couple has 2 children. Find the probability that both are boys given that the older child is a boy.

4. Given that the events A and B are such that \( P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5} \) \& \( P(B) = K \),

Then find “\( K \)” if A and B are independent.

5. If \( A \) and \( B \) are two independent events \( \text{ and } \ P(A) = 0.3, P(B) = 0.6 \), then find \( P(A \ & \ \text{not } B) \).

6. A car producing company knows from past experience that the probability of an order for cars will be ready for shipment on time is 0.85, and the probability that an order for cars will be ready for shipment and will be delivered on time is 0.75. What is the probability that an order for cars will be delivered on time given that it was ready for shipment on time?

7. A pair of dice is thrown and the product of the numbers is observed to be even. What is the probability that both dice have come up with even numbers?

8. A bag contains 4 yellow and 5 red balls and another bag contains 6 yellow and 3 red balls. A ball is drawn from the first bag and without seeing the colour, it is put in the second bag. A ball is drawn from the second bag. Find the probability that it is yellow in colour.

9. Consider the experiment of throwing a die, if a multiple of 3 come up throw the die again and if any other number comes toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 2'.

10. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A fair die is cast, if the face 1 or 3 turns up, a ball is taken from the first bag, and if any other face turns up a ball is chosen from the second bag. Find the probability of choosing a black ball.
11. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turn up a ball is picked up from bag B. If 4, 5 or 6 turn up a ball is picked up from bag C. Bag A contains 3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag picked up and a ball is drawn.

a) What is the probability of drawing a red ball? (b) If the ball drawn is red, what are the chances that bag B was picked up?

12. A bag contains 3 green and 7 white balls. Two balls are selected at random without replacement. If the second selected ball is given to be green what is the probability that the first selected ball is also green.

13. A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON?

14. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn successively. Also find the mean, variance and standard deviation of the distribution.

15. A bag contains 4 red and 5 black marbles. Find the probability distribution of number of red marbles in a random draw of three marbles. Also find the mean and standard deviation of the distribution.

16. In four throws of a pair of dice, what is the probability of throwing doublets (i) at least twice (ii) at most twice (iii) at least thrice (iv) not more than once.

17. If the probability of success is \( \frac{1}{100} \), then how many trials are necessary in order that the probability of getting at least one success is greater than half?
Value based questions

1. A man is known to speak truth 3 out of 4 times. He throws a die and reports it is a six. Find the probability that it is actually not a six? What value of life is violated here?

2. A, B and C visited a market for purchasing fruits. A purchased 1 kg. apples 3 kg. grapes and 4 kg. oranges and paid Rs. 800. B got 2 kg. apples, 1kg. grapes and 2 kg. oranges and paid Rs. 500, while C paid Rs 700 for 5 kg. apples, 1 kg. grapes and 1 kg. oranges. Find the cost of each fruit using matrix method. Why are fruits good for health?

3. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university a) none will graduate. (b) only one will graduate. (C) all will graduate. What do you think is the importance of education?

4. 70% men and 30% women are smokers. 10% of these men and 20% of these women use “X” brand. What is the probability that a person using “X” brand will be a man? Why should you discourage your friends from smoking?

5. Of the students in a school it is known that 60% come by public transport and 40% come by their own vehicle. Previous year result shows 30% of students who come by public transport report late to school and 40% of the students who come on their own, report late. At the end of the year, one student is chosen at random and is found to be late to school. What is the probability that he comes by public transport? In view of today’s depleting energy resources comment on the above.

6. A dietician has to develop a special diet using two food P and Q. Each packet (containing 30 gram) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of Vitamin A in the diet? What is the minimum amount of Vitamin A? Comment on the effects of cholesterol in the food?
1. At the point (2,1), find the slope of the curve \( x^6 y^6 = 64 \).

2. Find the derivative of \( \sin^{-1}(x^3) \).

3. Evaluate \( \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) \).

4. If \( c \) is a number that satisfies the conclusions of the Mean Value theorem for \( x^3 - 2x^2 \) on the interval \([0,2]\), find the value of \( c \).

5. If \( f(x) = \sqrt{9-x} \); \( g(x) = x^3 + 1 \), find \( f \circ g(x) \).

6. If \( f(x) = (x+1)e^x \), find the intervals in which the function is increasing.

7. Write the equation of the tangent to the curve \( x^3 - 3x + 2 \) at the point (2,4).

8. Find the stationary points of the function \( f(x) = (x-2)^2 \frac{2}{3}(2x+1) \).

9. Find the maximum value of the function \( f(x) = \sin 2x \) on the interval \([0,\frac{\pi}{2}]\).

10. If \( f(x) = x^4 \), defined from \( R \to R \), is this function one - one ?

11. If given that \( f(x) = 16x^2 + 8x - 14 \), is an invertible function, find its inverse.

12. Differentiate \( \cos(x^3) \) with respect to \( x^3 \).

13. Find the slope of the tangent to the curve represented by \( x = t^2 + 3t - 8; y = 2t^2 - 2t - 5 \) at (2,-1).

14. If \( y = \tan^{-1}\frac{4x}{1+5x^2} = \tan^{-1}\frac{2-3x}{3+2x} \), show that \( \frac{dy}{dx} = \frac{5}{1+25x^2} \).

15. Differentiate \( \log x \) with respect to \( e^x \).

16. Differentiate \( \tan^{-1}\frac{2x}{1-x^2} \) with respect to \( \sin^{-1}\frac{2x}{1+x^2} \).

17. If \( y = e^x + e^{2x} + e^{3x} + \ldots \infty \), prove that \( \frac{dy}{dx} = \frac{y}{1-y} \).

18. If \( y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots \infty}} \), prove that \( \frac{dy}{dx} = \frac{\sin x}{1-2y} \).
19. If \( y = \sqrt{x} + \frac{1}{\sqrt{x}} \), show that \( 2x \frac{dy}{dx} + y = 2\sqrt{x} \).

20. If \( y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right) \), show that \( \frac{dy}{dx} = 0 \).

21. Differentiate \( \tan^{-1}\left(\frac{1}{x^3} + \frac{1}{a^3}\right) \) with respect to “x”

22. If \( y = \sin^2 x^2 \), find \( \frac{dy}{dx} \).

23. If \( y = \sqrt{x+y} \), prove that \( \frac{dy}{dx} = \frac{1}{2y-1} \).

24. Find \( \frac{dy}{dx} \), if \( x = a \log t; y = b \sin t \).

25. Find \( \frac{dy}{dx} \), if \( x = \sqrt{\sin 2\theta}; y = \sqrt{\cos 2\theta} \).

26. If \( x = at^2, y = 2at \) find \( \frac{d^2y}{dx^2} \).

27. Show that the function \( f(x) = 2x + 3 \) is continuous at \( x = -4 \).

28. Show that the function \(|x-4|\) is a continuous function.

29. Show that the function \( f(x) = \begin{cases} \frac{x}{\sin 3x}, & x \neq 0 \\ 3, & x = 0 \end{cases} \) is discontinuous at \( x=0 \).

30. If the function \( f(x) = \begin{cases} \frac{\sin^2 kx}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases} \) is continuous at \( x=0 \), find “k”.

31. Show that the function \( f(x) = \sin|x| \) is a continuous function.

32. Show that the function \( f(x) = \frac{1}{x-5} \) is a continuous function.

33. If \( \tan^{-1}3 + \tan^{-1}x = \tan^{-1}8 \), then find \( x \).

34. Show that the function \( f(x) = \sin^2 x + x^2 - 2x \) is continuous at \( x=0 \).

35. Evaluate a) \( \cos^{-1}\left(\cos \frac{7\pi}{6}\right) \) b) \( \tan^{-1}\left(\tan \frac{3\pi}{4}\right) \)
36. Find the Principal value of $\cot^{-1}(-\sqrt{3})$.

37. Simplify $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$.

38. Find the value of a) $\cot\left(\tan^{-1}a + \cot^{-1}a\right)$ b) $\cos\left(\sec^{-1}x + \cos ec^{-1}x\right), |x| \geq 1$

39. Find the value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

40. The function $f(x) = \begin{cases} \sin3x, & x \neq 0 \\ \frac{x}{k}, & x = 0 \end{cases}$ is continuous at “$x = 0$”. Find “$k$”

41. Differentiate $\cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$ with respect to “$x$”

42. Differentiate $\tan^{-1}\left(\sqrt{1+x^2} - x\right), x \in R$ with respect to “$x$”

43. Differentiate with respect to “$x$” : $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$

44. Differentiate with respect to “$x$” : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

45. If $\sin y = x\sin(a+y)$, find $\frac{dy}{dx}$.

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**Practice assignment -II**

**Very Short Answer Type Questions**

Q1. Find the integrating factor for the following differential equation : $\frac{dx}{dy} - \frac{2x}{y} = 3y^3 - 5y + 1$

Q2. Show that the following differential equation is homogenous: $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$

Q3. If A, B are symmetric matrices and AB = BA, then show that AB is symmetric.

Q4. If A, B and AB are all symmetric matrices, then show that AB = BA.

Q5. If A, B are skew symmetric matrices and AB = BA, then show that AB is symmetric.

Q6. If A, B are square matrices of equal order and B is a skew symmetric matrix, then show that $ABA'$ is also skew symmetric.
Q7 If a matrix is both symmetric and skew symmetric, then show that it is a null matrix.

Q8 What is the number of all possible matrices of order 3×3 with each entry 0 or 1?

Q9 If A, B are square matrices of equal order and B is a symmetric matrix, then show that $A'B'A$ is also symmetric.

Q10 Give an example of two non-zero matrices A and B such that AB = O.

Q11 Give an example of two non-zero matrices A and B such that AB = O but BA ≠ O.

Q12 What is the order of AB + CB, where A, B and C are matrices of order 3×4, 4×2, 3×4 respectively.

Q13 Give an example of symmetric and skew symmetric matrix.

Q14 If $A=\begin{bmatrix} a_{ij} \end{bmatrix}$ is 3×3 matrix and $A_{ij}$'s denote the cofactors of the corresponding elements $a_{ij}$'s, then write the value of $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

Q15 If $A=\begin{bmatrix} a_{ij} \end{bmatrix}$ is 3×3 matrix and $A_{ij}$'s denote the cofactors of the corresponding elements $a_{ij}$'s, then write the value of $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$.

Q16 If $A=\begin{bmatrix} a_{ij} \end{bmatrix}$ is 3×3 matrix and $A_{ij}$'s denote the cofactors of the corresponding elements $a_{ij}$'s, then write the value of $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$.

Q17 If A is a square matrix of order 2 and $|A| = -5$, find the value of $|3A|$.

Q18 If A is a square matrix of order 3 and $|A| = -2$, find the value of $|5A|$.

Q19 If $x \in I$ and $\begin{bmatrix} 2x & 3 \\ -1 & x \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ x & -1 \end{bmatrix}$, find the value(s) of x.

Q20 Evaluate without expanding: $\begin{vmatrix} 2 & 2 & 2 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$

Q21 If A is a square matrix of order 3 such that $|adj A| = 100$, find $|A|$.

Q22 If A, B and C are all non-zero square matrices of the same order, then find the condition on A such that AB = AC implies B = C.

Q23 If A is a skew symmetric matrix of order 3, then show that $|A| = 0$.

Q24 Examine whether the following system of equations is consistent:

$2x - y = -2, 2y - z = -1, 3x - 5y = 3$.

Q25 Prove that the diagonal elements of a skew symmetric matrix are zero.

Q26 Find the values of x, y and z from the following equation:

$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$
Q27 Evaluate without expanding: \[ \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} \]

Q28 If \( A = \{-1,1,3\} \), then what is the number of relations on \( A \)?

Q29 Show that the function \( f : N \to N \), defined by \( f(x) = 2x - 1 \) is not onto.

Q30 Is the function \( f \) defined by \( f(x) = \begin{cases} 2x - 1, & x < 0 \\ x + 2, & x \geq 0 \end{cases} \) continuous at \( x = 0 \).

Q31 A four digit number is formed using the digits 1, 2, 3, 5 with no repetition. Find the probability that the number is divisible by 5.

Q32 Find the rate of change of the area of a circle with respect to its radius when the radius is 5 cm.

Q33 If \( A \) is a matrix of order \( 3 \times 4 \) then what should be the order of the matrix \( B \) such that \( A'B \) and \( BA' \) are both defined?

Q34 Evaluate: \[ \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} \]

Q35 Show that \( \cot^{-1} x + x \) is an increasing function on \( \mathbb{R} \).

Q36 Differentiate \( \cot(x^\cos x) \) w.r.t \( x^\cos x \).

Q37 Find the maximum and minimum value of \( 2 \sin x + 3 \cos x \)

Q38 If \( y = \sqrt{\cos x + y} \), find \( \frac{dy}{dx} \).

Q39 Find the value of \( x \) if \( \begin{vmatrix} 2 & x \\ 3 & 7 \end{vmatrix} = \begin{vmatrix} 2 & 8 \\ -1 & 7 \end{vmatrix} \)

Q40 Each side of an equilateral triangle is increasing at the rate of 8 cm/hr. Find the rate of increase of its area when side is 2 cm.

Q41 Find \( a \), for which \( f(x) = a(x + \sin x) + a \) is increasing.

Q42 What is the approximate change in the volume \( V \) of a cube of side \( x \) cm caused by increasing the side by 2%?

Q43 The diameter of a circle is increasing at the rate of 1 cm/sec. Find the rate of increase of its area when its radius is \( \pi \).
Q44 If the tangent to the curve \( x = at^2, y = 2at \) is perpendicular to x-axis, then find its point of contact.

Q45 Evaluate: \( \tan^{-1} \left( \frac{x}{y} \right) - \tan^{-1} \left( \frac{x-y}{x+y} \right) \)

Q46 Differentiate \( \sin^{-1} \left( \frac{1-x^2}{1+x^2} \right) \) w.r.t. \( x \).

Q47 If Rolle’s theorem is applicable to \( f(x) = e^x \sin x \) in \([0, \pi]\), then find the ‘c’ in Rolle's theorem.

Q48 If \( \cos (x-y) = \log (x-y) \), then find \( \frac{dy}{dx} \).

Q49 If the curve \( ay + x^2 = 7 \) and \( x^3 = y \) cut orthogonally at \((1, 1)\) then show that \( a = 6 \).

Q50 Write in the simplest form: \( \tan^{-1} \left( \frac{2\sqrt{x}}{1-x} \right) \)

Practice Assignment-III

Very Short Answer Type Questions

Q1 If a line makes angles \( \alpha, \beta, \gamma \) with the x,y,z axes respectively, find the value of \( \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma \).

Q2 Evaluate, \( \int_{0}^{1.5} \lfloor x \rfloor \, dx \) (where \( \lfloor x \rfloor \) is greatest integer function)

Q3 Evaluate, \( \int_{0}^{1.5} \lfloor x^2 \rfloor \, dx \) (where \( \lfloor x \rfloor \) is greatest integer function)

Q4 Write the order and degree of the differential equation, \( y = x \frac{dy}{dx} + a \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \)

Q5 If \( f(1) = 4; f'(1) = 2 \), find the value of the derivative of \( \log f(e^x) \) w.r.t \( x \) at the point \( x = 0 \).

Q6 Let \( f : R - \left\{ \frac{-3}{5} \right\} \rightarrow R \) be a function defined as \( f(x) = \frac{2x}{5x + 3} \), find \( f^{-1} \) : Range of \( f \rightarrow R - \left\{ \frac{-3}{5} \right\} \)

Q7 For any three vectors \( \vec{a}, \vec{b}, \vec{c} \), write the value of \( \vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) \).

Q8 Find vector equation of line through points with position vectors \( 2\hat{i} - 3\hat{j} + \hat{k} \) and \( \hat{i} + 3\hat{j} - \hat{k} \).

Q9 What should be the angle between vectors \( \vec{a} \) and \( \vec{b} \) such that \( |\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \).
Q10 Write a value of \( \int \frac{1 + \cot x}{x + \log \sin x} \, dx \)

Q11 Show that the differential equation \( 2ye^{y/2} \, dx + (y - 2xe^{y/2}) \, dy = 0 \) is homogenous.

Q12 If \( f'(x) = \frac{4}{x^2} \) and \( f(1) = 6 \), find \( f(2) \)

Q13 If \( f(x) = e^x g(x) \), \( g(0) = 2 \) and \( g'(0) = 1 \), then find \( f'(0) \).

Q14 What are the maximum and minimum values of \( 3\sin x + 4 \cos x \)?

Q15 Let \( f \) and \( g \) be differentiable functions satisfying
\[
g'(a) = 2, g(a) = b \text{ and } f \circ g = I \text{(identity function)}, \text{show that } f''(b) = \frac{1}{2}.
\]

Q16 If \( y = \cos^{-1} \left( \frac{2 \cos x - 3 \sin x}{\sqrt{13}} \right) \), then show that \( \frac{dy}{dx} = 1 \).

Q17 Write a value of \( \int (\cos(\log x) + \sin(\log x)) \, dx \)

Q18 If \( f(x) = \frac{|x|}{x}, x \neq 0 \), show that \( |f(\alpha) - f(-\alpha)| = 2 \), where \( \alpha \neq 0 \)

Q19 If \( f(x) = \left( a - x^n \right)^{1/n} \), then find \((f \circ f)(x)\)

Q20 Write the order and degree of differential equation \( x \frac{dy}{dx} + \frac{3}{y} = y^2 \)

Q21 Let \( f : \mathbb{R} \to \mathbb{R} \) be a mapping defined by \( f(x) = x^3 + 5 \), find \( f^{-1} : \mathbb{R} \to \mathbb{R} \)

Q22 If \( f(x) = \sin x \) and \( g(x) = \cos x \), find \( (2f)(\frac{\pi}{2}) \) and \( (f - g)(\frac{\pi}{2}) \)

Q23 Write a vector of magnitude 15 units in the direction of \( 2\hat{i} + 4\hat{j} - 5\hat{k} \)

Q24 For any \( \vec{r} \), find \( (\vec{r} \cdot \hat{i})\hat{i} + (\vec{r} \cdot \hat{j})\hat{j} + (\vec{r} \cdot \hat{k})\hat{k} \)

Q25 If a unit vector \( \vec{a} \) makes angles \( \frac{\pi}{3} \) with \( \hat{i} \), \( \frac{\pi}{4} \) with \( \hat{j} \), and an acute angle \( \theta \) with \( \hat{k} \), then find the value of \( \theta \).

Q26 If \( \vec{a} \) and \( \vec{b} \) are two vectors of magnitude 3 and \( 2/3 \) respectively such that \( \vec{a} \times \vec{b} \) is a unit vector, find the angle between \( \vec{a} \) and \( \vec{b} \).
Q27 Find $x$ if $\left(\vec{x} - \vec{a}\right) \cdot \left(\vec{x} + \vec{a}\right) = 75$, where $\vec{a}$ is a unit vector.

Q28 Find a solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$

Q29 Find the integrating factor of differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$

Q30 Write the order and degree of differential equation $5 \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{1}{2}}$

Q31 Find the differential equation corresponding to $y = Ae^x + Be^{-x}$

Q32 Let $f : R \rightarrow R$ be given by $f(x) = x^2 - 3$. Find $f^{-1} : R \rightarrow R$.

Q33 Find the slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1).

Q34 Write the position vector of a point dividing the line segment joining points A and B whose position vectors are $2\hat{i} - 3\hat{j} + 5\hat{k}$ and $3\hat{i} + 2\hat{j} - \hat{k}$ in the ratio 1:4 externally.

Q35 If $\theta = \sin^{-1} \left(\sin(-600^\circ)\right)$, then find one of the possible values of $\theta$.

Q36 If $f : [2, \infty[ \rightarrow X$ defined by $f(x) = 4x - x^2$ is given to be invertible, then find X.

Q37 Evaluate: $\sin \left[\frac{\pi}{3} - \sin^{-1} \left(\frac{-1}{2}\right)\right]$

Q38 If $y = \log \sqrt{\tan x}$, then find $\frac{dy}{dx}$.

Q39 If $y = \sin^{-1} \left(\frac{1 - x^2}{1 + x^2}\right)$ then find $\frac{dy}{dx}$.

Q40 If $x = a \cos nt - b \sin nt$, then find $\frac{d^2x}{dt^2}$.

Q41 If the line $y = x$ touches the curve $y = x^2 + bx + c$ at (1, 1), then show that $b = -1$ and $c = 1$.

Q42 If the curves $y = ae^x$ and $y = be^{-x}$ cut orthogonally, then show that $ab = 1$.

Q43 If $\tan^{-1} \left(\cot \theta\right) = 2\theta$, then find a possible value of $\theta$.

Q44 Evaluate $\cos^{-1} \left(\cos \frac{5\pi}{3}\right) + \sin^{-1} \left(\sin \frac{5\pi}{3}\right)$

Q45 Find a unit vector parallel to the sum of $5\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 6\hat{j} + \hat{k}$

Q46 Find the vector projection of $3\hat{i} - \hat{j} + 5\hat{k}$ on $-2\hat{i} + 3\hat{j} + \hat{k}$. 
Q47 Find a vector which is equally inclined to the axes.

Q48 Find the value $\lambda$ of if $22\hat{i} - 3\hat{j} + 5\hat{k}$ is perpendicular to $2\hat{i} + \lambda\hat{j} + \hat{k}$.

Q49 If $\vec{a}$ and $\vec{b}$ are unit vectors such that $|\vec{a} + \vec{b}| = 1$, then find $|\vec{a} - \vec{b}|$.

Q50 If $\sin^{-1}x - \cos^{-1}x = \pi/6$, then find $x$.

Q51 Find $P(A \cap B)$, if $P(B) = \frac{5}{13}$ and $P(A \mid B) = \frac{2}{5}$.

Q52 If $P(A) = 0.2, P(B) = p, P(A \cup B) = 0.6$ and A and B are given to be independent events, then find “p”.

Q53 If A and B are two independent events and $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$, then find $P(A^C \cap B^C)$.

Q54 If $P(A) = \frac{1}{2}, P(B) = p, P(A \cup B) = \frac{3}{5}$, then find “p”, if A and B are mutually exclusive.

Q55 A coin is tossed thrice and all eight outcomes are assumed to be equally likely. Are the following events independent.

$E_1 =$ The first throw results in a head. $E_2 =$ the last throw results in tail

Q56 If $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{8}$, then find $P$(not A and not B).

Q57 If A and B are two events such that $P(A) = 0.4, P(B) = 0.8$ & $P(B \mid A) = 0.6$, then find $P(A \mid B)$.

Q58 In a class 40% study Mathematics, 25% study Biology and 15% study both Mathematics and Biology. One student is selected at random. Find the probability that he studies Mathematics given that he studies Biology.

Q59 A five - digit number is formed using the digits 1,2,3,4,5 with no repetition. Find the probability that the number is an even number.

Q60 If $P(A \cup B) = 0.8, P(A \cap B) = 0.3$, then find $P(A) + P(B)$.

Q61 A coin is tossed three times, if head occurs on first two tosses, find the probability of getting head on third toss.

Q62 A fair die is rolled. Consider the events $A = \{1,3,5\}$ & $B = \{2,3\}$. Find $P(A \mid B)$.

Q63 A die is rolled twice and the sum of the numbers appearing is observed to be 4. Find the probability that the number 1 has appeared once.

Q64 A couple has 2 children. Find the probability that both are boys given that the older child is a boy.

Q65 Two coins are tossed. What is the probability of two heads coming up if it is known that at least one head come up.

Q66 A and B are two events such that $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$ & $P(A \cup B) = \frac{7}{11}$, then find $P(B \mid A)$. 

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Class XII / Mathematics/41
Q67  Given that the events A and B are such that \( P(A) = \frac{1}{2} \), \( P(A \cup B) = \frac{3}{5} \) \& \( P(B) = K \), Then find “K” if A and B are independent.

Q68  Two dice are thrown. Find the probability of getting an odd number on the first die and a multiple of 3 on the other die.

Q69  If A and B are two independent events and \( P(A) = 0.3, P(B) = 0.6 \), then find \( P(A \& \text{not} B) \).
Practice Paper for Summer Vacation

Time: 3hrs.  MM: 100

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 26 questions divided into three sections A, B and C. Section A contains 6 questions of 1 mark each, section B is of 13 questions of 4 marks each and section C is of 7 questions of 6 marks each.
3. All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, an internal choice has been provided in two questions of 4 marks each and two questions of 6 marks each.
5. There are 4 printed sides in this paper.

Section A

1. Evaluate: \( \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) \)
2. Let \( f \) be the exponential function and \( g \) be the logarithmic function. Find \( (fog)(1) \)
3. If \( 4\sin^{-1}x + \cos^{-1}x = \pi \), then find the value of \( x \).
4. Differentiate \( 2\sqrt{\cos\sqrt{x}} \) w.r.t \( x \).
5. Relation \( R \) in the set of natural numbers is defined by: \( R = \{(x, y) : x, y \in N, x \leq y^2\} \). Is \( R \) transitive?
6. If \( P(A) = 0.2, P(B) = p, P(A \cup B) = 0.6 \) and \( A \) and \( B \) are given to be independent events, then find “\( p \)”

Section B

7. Show that the function \( f : R \rightarrow R \) defined by \( f(x) = \frac{3x + 1}{2} \) is invertible. Also find the inverse of \( f \).

OR

Prove that the relation \( R \) on the set \( Z \) of all integers defined by \( (x, y) \in R \iff x-y \) is divisible by \( n \) is an equivalence relation on \( Z \).
8. Let * be a binary operation defined on \( \mathbb{N} \times \mathbb{N} \), by \((a,b) * (c,d) = (a+c,b+d)\). Show that * is commutative and associative. Also, find the identity element for * on \( \mathbb{N} \times \mathbb{N} \), if any.

9. Prove that: 
\[
\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2},\quad 0 < x < \frac{\pi}{2}
\]

10. Prove: 
\[
\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} + \tan^{-1}\frac{8}{19} = \frac{\pi}{4}
\]

11. If \( y = x^{\cos x} + \frac{x^2 + 1}{x^2 - 1} \), find \( \frac{dy}{dx} \).

12. If \( f: [0, \infty) \to \mathbb{R} \) & \( g: \mathbb{R} \to \mathbb{R} \), defined by \( f(x) = \sqrt{x} \) & \( g(x) = -x^2 - 1 \), then find \( f \circ g \) & \( g \circ f \), if they exist.

13. Prove that: 
\[
\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi.
\]

14. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

15. Find the value of \( x \) if: 
\[
\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}
\]

16. If \( \cos y = x \cos(a + y) \), show that 
\[
\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}
\]

17. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that
(i) the youngest is a girl.
(ii) Atleast one is a girl.

18. Differentiate w.r.t \( x \): 
\[
(\log x)^x + x^{\log x}
\]

19. Differentiate \( \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) \), \( -1 \leq x \leq 1 \) w.r.t \( x \).

OR

Find \( \frac{dy}{dx} \), if \( x^3 + x^2 y + xy^2 + y^3 = 0 \).
20. If \( x = a(\theta + \sin \theta); \ y = a(1 - \cos \theta) \), prove that \( \frac{d^2y}{dx^2} = \frac{1}{a} \) at \( \theta = \frac{\pi}{2} \)

21. Differentiate \( \tan^{-1}\left(\frac{x}{1 + \sqrt{1 - x^2}}\right) \) with respect to \( \sec^{-1}\left(\frac{1}{2x^2 - 1}\right) \).

OR

Differentiate \( \cos^{-1}\left[\frac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right] \) w.r.t. \( \tan^{-1}\left[\frac{x}{1 + \sqrt{1 - x^2}}\right] \).

22. Show that the function \( f \) defined as follows, is continuous at \( x = 2 \), but not differentiable there at:

\[
f(x) = \begin{cases} 
3x - 2, & 0 < x \leq 1 \\
2x^2 - x, & 1 < x \leq 2 \\
5x - 4, & x > 2
\end{cases}
\]

23. Consider the binary operation \( * : R \times R \rightarrow R \) and \( \circ : R \times R \rightarrow R \) defined as \( a * b = |a - b| \) and \( a \circ b = a, \forall a, b \in R \). Show that \( * \) is commutative but not associative, \( \circ \) is associative but not commutative. Further show that \( * \) distributes over \( \circ \) but converse is not true.

OR

Consider the function \( f : R_+ \rightarrow [9, \infty) \) defined by \( f(x) = x^2 + 9, \) where \( R_+ \) is the set of all non-negative real numbers. Show that \( f \) is invertible, also find inverse of \( f \).

24. If \( \sqrt{1 - x^4} + \sqrt{1 - y^4} = a(x^2 - y^2) \), show that \( \frac{dy}{dx} = \frac{x}{y} \sqrt{\frac{1 - y^4}{1 - x^4}} \)

OR

If \( x\sqrt{1+y} + y\sqrt{1+x} = 0 \), show that \( \frac{dy}{dx} = \frac{-1}{(1+x)^2} \)
25. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence find the mean of the distribution.

26. Three bags contain balls as shown in the table below:

<table>
<thead>
<tr>
<th>Bag</th>
<th>No. of White balls</th>
<th>No. of Black balls</th>
<th>No. of Red balls</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

A bag is chosen at random and two balls are drawn from it. They happened to be white and red. What is the probability that they came from Bag III?
Practice Test-1

Relations And Functions

Q1 to Q6 carry 1 mark each, Q7 to Q12 carry 4 marks each. M.M.30

Q1. Give an example of a relation which is symmetric and reflexive but not transitive.

Q2. Check the injectivity of \( f: \mathbb{Z} \rightarrow \mathbb{Z} \) given by \( f(x) = x^2 \).

Q3. If \( f = \{(1,3),(2,7),(8,6)\} \) and \( g = \{(7,11),(6,0),(3,5)\} \), find \( gof \).

Q4. Let \( f: \mathbb{R} \rightarrow \mathbb{R} \) be given by \( f(x) = (3-x^3)^{\frac{1}{3}} \), then find \( fof(x) \).

Q5. If \( a*b = \frac{a+2}{3b} \), find \( (5*3)*1 \) where \(*\) is a binary operation on \( \mathbb{R} \).

Q6. Determine if \(*\) associative for the binary operation \(*\) defined as \( a*b = 2^{ab} \) on \( \mathbb{Z}^+ \).

Q7. Consider \( f: R_+ \rightarrow [-5, \infty) \) given by \( f(x) = 9x^2 + 6x - 5 \). Show that \( f \) is invertible. Find the inverse of \( f \).

Q8. Let \( R \) be a relation on the set \( A \) of ordered pairs of positive integers defined by \( (x, y) R (u, v) \) if and only if \( xy = uv \). Show that \( R \) is an equivalence relation.

Q9. Let \( X \) be a non-empty set. \( P(X) \) be its power set. Let \( \circ \) be an operation defined on elements \( P(x) \) by, Then, prove that \( \circ \) is a binary operation in \( P \). Is \( \circ \) commutative? Is \( \circ \) associative?
Find the identity element in \( P(X) \) w. r. t \( \circ \). Find all the invertible elements of \( P(X) \).
If \( \odot \) is another binary operation defined on \( P(X) \) as \( A \odot B = A \cup B \), then verify that \( \odot \) distributes itself over \( \circ \).

Q10. Show that the function \( f: R \rightarrow R \) defined by \( f(x) = \frac{3x+1}{2} \) is one-one and onto. Hence find the inverse of the function.

Q11. Show that the function \( f: A \rightarrow B \) defined as \( f(x) = \frac{3x+4}{5x-7} \), where \( A = R - \frac{7}{5} \), \( B = R - \frac{3}{5} \) is invertible and hence find \( f^{-1} \).

Q12. Let \( * \) be a binary operation on \( Q \), such that \( a * b = a + b - ab \).
   (i) Prove that \( * \) is commutative and associative.
   (ii) Find identity element of \( * \) in \( Q \) (if it exists).
Inverse Trigonometric Functions

Q1 to Q6 carry 1 mark each, Q7 to Q12 carry 4 marks each. M.M: 30

Q1. Write the value of $\tan\left(2 \tan^{-1} \frac{1}{5}\right)$.

Q2. Write the value of $2 \cos^{-1} \frac{1}{2} + 3 \sin^{-1} \frac{1}{2}$.

Q3. Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \left(-\frac{\pi}{3}\right)\right)$.

Q4. Solve for $x$: $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$.

Q5. Evaluate: $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.

Q6. Evaluate: $\cot^{-1}\left[2 \sin\left(2 \cos^{-1} \frac{1}{2}\right)\right]$.

Q7. Solve for $x$: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$.

Q8. Prove that: $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \frac{\pi}{4} - \frac{x}{2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Q9. Prove that: $\tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$.

Q10. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \left(\frac{\pi}{2} - \frac{1}{2} \cos^{-1} x\right)$.

Q11. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

Q12. Show that: $\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$.
Practice Test 3
Matrices and Determinants

Q1 to Q4 carry 1 mark each, Q5 to Q9 carry 4 marks each and Q10 carry 6 marks each.
M.M: 30

Q1. If \[
\begin{vmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{vmatrix}
\] = 0, find the value of x.

Q2. Find A if \[A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\] and \[2A + B = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\]

Q3. If \[A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\] satisfies \[A^4 = \lambda A\], find the value of \(\lambda\).

Q4. If A is a square matrix of order 3 and \[|A| = -4\], find the value of \[|4A|\].

Q5. Express \[\begin{bmatrix} 1 & 0 & 2 \\ 4 & 2 & 5 \end{bmatrix}\] as a sum of symmetric and skew symmetric matrices.

Q6. If \[\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 3 \end{bmatrix}\], prove that \[A^3 - 6A^2 + 7A + 2I = 0\].

Q7. Using elementary transformations find the inverse of \[\begin{bmatrix} 3 & 1 \\ 6 & 4 \end{bmatrix}\].

Q8. By using properties of determinants show that:
\[
\begin{vmatrix}
1 & x & y \\
x & x^2 & yz \\
y & y^2 & xz
\end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)
\]

Q9. By using properties of determinants show that:
\[
\begin{vmatrix}
3a & -a + b & -a + c \\
-b + a & 3b & -b + c \\
-c + a & -c + b & 3c
\end{vmatrix} = 3(a + b + c)(ab + bc + ca)
\]

Q10. An amount of Rs 600 crores is spent by the government in three schemes. Scheme A is for saving girl child. Scheme B is for saving of newly wed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Twice the amount spent on Scheme C together with amount spent on Scheme A is Rs 700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is Rs 1200 crores. Find the amount spent on each Scheme using matrices? What is the importance of saving girl child? Suggest any one measure that you will do to save girl child.
Q1. If \( f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x + 1}, & x \neq -1 \\ \lambda, & x = -1 \end{cases} \) is continuous at \( x = -1 \), find the value of \( \lambda \).

Q2. Differentiate \( \sin^{-1}\left(\frac{2x}{1+x^2}\right) \) w.r.t. \( \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \).

Q3. If \( \log\left(\sqrt{1+x^2} - x\right) = y\sqrt{1+x^2} \) show that \( (1+x^2)\frac{dy}{dx} + xy + 1 = 0 \).

Q4. Find the value of “\( a \)” for which the function defined as
\[
 f(x) = \begin{cases} 
 a \sin \frac{\pi}{2} (x+1), & x \leq 0 \\
 \tan x - \sin x \overline{x^3}, & x \geq 0 
\end{cases}
\]
is continuous at \( x = 0 \).

Q5. Differentiate \( x^\cos x + \frac{x^2 + 1}{x^2 - 1} \) w.r.t. \( x \).

Q6. If \( x^y = y^x \), show that \( \frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)} \).

Q7. Find \( \frac{dy}{dx} \) if \( y = \sin^{-1}\left[ x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] \).

Q8. If \( \cos^{-1}\left[\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right] \) find \( \frac{dy}{dx} \).

Q9. If \( x = a(\cos t + t \sin t) \) and \( y = b(\sin t - t \cos t) \), find \( \frac{d^2y}{dx^2} \) at \( t = \frac{\pi}{3} \).

Q10. Verify Lagrange’s Mean Value Theorem for \( f(x) = x(x-1)(x-2) \) in \( [0, \frac{1}{2}] \).

Q11. If \( x = \sin t \) and \( y = \sin pt \), prove that \( (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0 \).

Q12. Differentiate \( \cos^{-1}\left[\frac{\sqrt{1-x^2} + x}{\sqrt{2}}\right] \) w.r.t \( \tan^{-1}\left[\frac{x}{\sqrt{1-x^2} + 1}\right] \).

Q13. If \( \cos y = x \cos(a + y) \), prove that \( \frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a} \).

Q14. If \( y = \left(\tan^{-1} x\right)^2 \), prove that \( (1+x^2)^2\frac{d^2y}{dx^2} + 2x\left(1+x^2\right)\frac{dy}{dx} = 2 \).
Practice Test 5

Application of Derivatives

Q1&Q2 are for 1 mark each, Q3 to Q11 are for 4 marks each, Q12 & 13 are for 6 marks each.

MM: 50

Q1. Use differential to approximate \( \sqrt{0.0037} \).

Q2. Find the approximate change in the volume of a cube of side \( x \) meters caused by increasing the side by 1%.

Q3. Find the intervals in which \((x-1)^3(x-2)^2\) is increasing and decreasing.

Q4. Find the equation of tangent and normal to the curve \( x = a \sin^3 t, y = b \cos^3 t \) at the point \( t'' \).

Q5. Find equation(s) of tangent drawn to the curve \( y^2 - 2x^3 - 4y + 8 = 0 \) from the point (1,2).

Q6. Find the intervals in which \( f(x) = 2 \log(x-2) - x^2 + 4x + 1 \) is increasing and decreasing.

Q7. Find all points of local maxima and minima and corresponding maximum and minimum values of the function \( f(x) = -\frac{3}{4} x^4 - 8x^3 - \frac{45}{2} x^2 + 105 \).

Q8. Find points of local maxima and minima of \( f(x) = \sin x + \frac{1}{2} \cos 2x \) where \( 0 \leq x \leq \frac{\pi}{2} \). Also find the absolute maximum and minimum values of the function.

Q9. Water is dripping out from a conical funnel at a uniform rate of 4 \( \text{cm}^3/\text{sec} \) through a tiny hole at the vertex in the bottom. When the slant height of water is 3 cm, find the rate of decrease of the slant height of the water cone given that the vertical angle of the funnel is 120°.

Q10. Sand is being poured into a conical pile at a constant rate of 50 \( \text{cm}^3/\text{sec} \) such that the height of the cone is always one half of the radius of the base. How fast is the height of the pile increasing when the sand is 5 cm deep?

Q11. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum when the angle between them is \( \frac{\pi}{3} \).

Q12. Show that the volume of the largest cone that can be inscribed in a sphere of radius ‘\( r \)’ is \( \frac{8}{27} \) of the volume of the sphere.

Q13. An open box with a square base is to be made out of a given quantity of cardboard of area \( c^2 \text{ sq.units} \). Show that the maximum volume of the box is \( \frac{c^3}{6\sqrt{3}} \) cu. units.
Practice Test 6

Integration

Q1 to Q8 are for 4 marks each. Q9 to Q11 are for 6 marks each. MM: 50

Evaluate the following:

Q1. \[\int \frac{x^2 - 3x}{(x-1)(x-2)} \, dx\]

Q2. \[\int \frac{\sin x}{\sin 4x} \, dx\]

Q3. \[\int e^{2x} \cos^2 x \, dx\]

Q4. \[\int \frac{x^2}{(x^2+4)(x^2+9)} \, dx\]

Q5. Prove that \[\int_0^1 \cot^{-1} \left(1 - x + x^2\right) \, dx = \frac{\pi}{2} - \log 2\]

Q6. \[\int x\sqrt{1 + x - x^2} \, dx\]

Q7. \[\int \frac{dx}{5 + 7 \cos x + \sin x}\]

Q8. \[\int_a^b \frac{a - x}{a + x} \, dx\]

Q9. \[\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} \, dx\]

Q10. \[\int_0^1 (3x^2 + 2x + 1) \, dx\] as a limit of sums.

Q11. \[\int \frac{dx}{\cos x(5 - 4 \sin x)}\]
Practice Test 7

Application of Integration and Differential Equations

Q1 to Q8 are for 4 marks each and Q9 to Q11 are for 6 marks each. MM: 50

Q1. Find the differential equation of all circles touching the x axis at the origin.
Q2. Find the differential equation of all circles in the first quadrant which touch the coordinate axes.
Q3. Solve:\n\( \frac{dy}{dx} = x + 1 \), \( y(0) = 5 \)
Q4. Solve:\n\( x(dy - ydx) = ydx \)
Q5. Solve the differential equation:\n\( (x + y)^2 \frac{dy}{dx} = a^2 \)
Q6. Find the particular solution of the equation:\n\( (x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0, y(1) = 1 \)
Q7. Solve the differential equation:\n\( (1 + y^2)dx = (\tan^{-1} y - x)dy \)
Q8. Find the particular solution of the equation:\n\( ye^y dx = (y^3 + 2xe^y)dy, y(0) = 1 \)
Q9. Find the area of the region \( \{ (x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9 \} \)
Q10. Find the area of the region bounded by the curves \( y = x^2 + 2, y = x, x = 0 \) and \( x = 1 \)
Q11. Find area of the region common to \( x^2 + y^2 = 16 \) and \( 6y = x^2 \)
Practice Test 8
Vectors and 3- Dimensional Geometry

Q1 to Q8 are for 4 marks each and Q9 to Q11 are for 6 marks each. MM: 50

Q1. If \( \mathbf{a} = \hat{i} + \hat{j} + \hat{k} \), \( \mathbf{b} = 4\hat{i} - 2\hat{j} + 3\hat{k} \) and \( \mathbf{c} = \hat{i} - 2\hat{j} + \hat{k} \); find a vector of magnitude 6 units which is parallel to \( 2\mathbf{a} - \mathbf{b} + 3\mathbf{c} \).

Q2. If \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) are three vectors of magnitude 3,4,5 respectively such that each is perpendicular to the sum of the other two, prove that \( \mathbf{a} + \mathbf{b} + \mathbf{c} = 5\sqrt{2} \).

Q3. Find the equation of the plane passing through the points (3,2,1) and (0,1,7) and parallel to the line \( \frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-1}{-1} \).

Q4. Find equation of the perpendicular drawn from the point (2,4,-1) to the line \( \frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} \).

Q5. The dot products of a vector with the vectors \( \hat{i} - 3\hat{k} \), \( \hat{i} - 2\hat{k} \) and \( \hat{i} + \hat{j} + 4\hat{k} \) are 0,5 and 8 respectively. Find the vector.

Q6. If \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) are three unit vectors such that \( \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} = 0 \) and angle between \( \mathbf{b} \) and \( \mathbf{c} \) is \( \frac{\pi}{6} \) prove that, \( \mathbf{a} = \pm 2(\mathbf{b} \times \mathbf{c}) \).

Q7. Let \( \mathbf{a} = \hat{i} - \hat{j} \), \( \mathbf{b} = 3\hat{j} - \hat{k} \) and \( \mathbf{c} = 7\hat{i} - \hat{k} \); find a vector \( \mathbf{d} \) which is perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \) and \( \mathbf{c} \cdot \mathbf{d} = 1 \).

Q8. Find the coordinates of the point where the line through \( (3, -4, -5) \) and \( (2, -3, 1) \) crosses the plane determined by the points \( (1,2,3), (2,2,1) \) and \( (-1,3,6) \).

Q9. Find the equation of the plane through the intersection of the planes \( \mathbf{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \), \( \mathbf{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0 \), and which is perpendicular to the plane \( \mathbf{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0 \).

Q10. Find the distance of the point \( (-1,-5,-10) \) from the point of intersection of the line \( \mathbf{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda (3\hat{i} + 4\hat{j} + 2\hat{k}) \) and plane \( \mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) - 5 = 0 \).

Q11. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point \( (3,2,1) \) from the plane \( 2x - y + z + 1 = 0 \). Also find the image of the point in the plane.
Q1. A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the two bags is selected and a ball is drawn. If it is found to be red, find the probability the second bag was chosen.

Q2. The probability of A, B and C solving a problem independently is $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{3}$. If all of them try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly two of them will solve (iii) at most two of them solve.

Q3. There are three urns containing A, B and C. A contains 10 red and 4 green marbles. B contains 9 red and 5 green marbles. C contains 8 red and 6 green marbles. One ball is drawn from each of these urns. What is the probability that, out of these three balls drawn, two are red and one is green?

Q4. Rohan and Sid throw a die alternatively till one of them gets a 6 and wins the game. Find the probability of Sid winning the game if Rohan starts the game.

Q5. There are two bags. The first bag contains 5 white and 6 black marbles. The second bag contains 4 white and 7 black marbles. Two balls are drawn from the first bag and without noticing their colour, are put into the second bag. Then two balls are drawn from the second bag. What is the probability that the balls drawn are white?

Q6. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that a 4 has occurred. What is the probability that a 4 has actually occurred?

Q7. A card from a deck of 52 cards is lost. From the remaining cards two cards are drawn and found to be both spades. What is the probability that a spade card is lost?

Q8. The probability of a man hitting a target is $\frac{1}{2}$. How many times must he fire so that the probability of hitting the target at least once is more than 90%.

Q9. From a lot of 12 items containing 3 defective, a sample of 3 items is drawn. Let $X$ denote the number of defective items drawn. Find the mean of $X$.

Q10. In a game, a man wins rupees five for a six and loses rupee one for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

Q11. An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.
Sample Paper I

General Instructions:

a. All questions are compulsory.
b. The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of 4 marks each and section C comprises of 7 questions of six mark each.
c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d. Internal choice has been provided in some questions. You have to attempt only one of the choices in such questions.
e. Use of calculators is not permitted. You may ask for logarithmic tables, if required.
f. This paper consists of 4 printed sides.

Section A

1. Find the value of x and y if
   \[
   \begin{bmatrix}
   x + y & -2 \\
   6 & 4
   \end{bmatrix} =
   \begin{bmatrix}
   4 & -2 \\
   2x & 5
   \end{bmatrix}
   \]

2. Evaluate
   \[\int \frac{x^2}{1 + x^3} dx\]

3. Find a value of
   \[\int \sqrt{3x^2 - 16} dx\]

4. Construct a \(2 \times 2\) matrix \(\begin{bmatrix} a_{ij} \end{bmatrix}\), where
   \[a_{ij} = \frac{(2i + j)^2}{3}\]

5. Given a square matrix A of order \(3 \times 3\) such that \(|A| = -2\), find the value of \(2A \text{ adj}A\)

6. Find the value of \(\lambda\) if the line
   \[\frac{x - 2}{9} = \frac{y - 1}{\lambda} = \frac{z + 3}{-6}\]
   is perpendicular to the plane
   \[3x - y - 2z = 7\]

Section B

7. Solve:
   \[ydx + x \log \frac{y}{x} dy - 2x dy = 0\]

8. Prove using properties of determinants,
   \[\begin{vmatrix}
   x & y & z \\
   x^2 & y^2 & z^2 \\
   yz & xz & xy
   \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)\]
9. If \( y = \sin^{-1} \sqrt{x} + (\sin x)^2 \), find \( \frac{dy}{dx} \).

10. The sum of the perimeter of a circle and a square is “k”, where k is a constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.

11. Find the value of \( \lambda \), if the lines \( \frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11} \) and \( \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \) are perpendicular to each other.

12. A and B throw a pair of dice alternately till one of them gets the sum as 6 and wins the game. Find their respective probabilities of winning if A starts the game

(or)

An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in a random draw of three balls.

13. If \( y = x^e \), prove that \( \frac{d^2 y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0 \).

14. Evaluate \( \int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \).

(or)

Prove that \( \int_{0}^{1} \frac{1-x}{\sqrt{1+x}} dx = \frac{\pi}{2} - 1 \).

15. Solve the following differential equation: \( ye^y dx = (y^3 + 2xe^y) dy \), \( y(0) = 1 \)

16. Evaluate: \( \int \frac{dx}{\cos^4 x + \sin^4 x} \)

17. Find the equation of the tangent and normal to the curve

\[ x = 1 - \cos \theta; \quad y = \theta - \sin \theta \] at \( \theta = \frac{\pi}{4} \)

18. Prove that \( \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} = \frac{\pi}{2} \)
19. Discuss the continuity of the function \( f(x) = \begin{cases} 
1, & 0 \leq x < \frac{1}{2} \\
\frac{1}{2}, & x = \frac{1}{2} \\
1 - x, & \frac{1}{2} < x \leq 1 
\end{cases} \)

(or)

Verify Rolles theorem for the function \( f(x) = \sin x + \cos x \), in the interval \( \left[ 0, \frac{\pi}{2} \right] \)

Section C

20. Find the distance of the point (3,4,5) from the plane \( x + y + z = 2 \) measure parallel to the line \( 2x = y = z \)

21. Using integration find the area of the region bounded by the lines \( 2x + y = 4, \ 3x - 2y = 6 \) and \( x - 3y + 5 = 0 \)

(OR)

Using integration, find the area of the region given by \( \{(x, y) : 9x^2 + y^2 \leq 36 \text{ and } 3x + y \geq 6\} \)

22. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulbs produce respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and is found to be defective. Find the probability that the bulb was produced by the machine A.

23. Define a binary operation \( * \) on the set \( A = \{0, 1, 2, 3, 4, 5\} \) given by \( a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\
a + b - 6, & \text{if } a + b \geq 6 \end{cases} \). Find the identity element of this operation and show that each element of the set is invertible and find the inverse.

24. A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are priced at Rs. 100 and Rs. 120 per unit respectively, how should the producer use his resources to maximize the total revenue? Solve the problem graphically.
25. The dot products of a vector with the vectors $\hat{i} - 3\hat{k}, \hat{i} - 2\hat{k}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. Find the vector. Also, find the unit vector in the direction of this vector obtained.

26. Find $A^{-1}$, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ and hence solve the following system of linear equations

\[
\begin{align*}
  x + 2y + 3z &= -4 \\
  2x + 3y - 3z &= 2 \\
  -3x + 2y - 4z &= 11
\end{align*}
\]
Sample Paper 2

Time : 3Hours

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of 4 marks each and section C comprises of 7 questions of six mark each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. Internal choice has been provided in some questions. You have to attempt only one of the choices in such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.
6. This paper consists of 4 printed sides.

Section A

1. Let \( f : R \rightarrow \left\{ \frac{5}{2} \right\} \) be an onto function defined as \( f(x) = \frac{4x}{2x+5} \), then find \( A \)

2. If \( A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \), then find the value of \( |(adjA)| \)

3. Using determinants find “k”, if the area of a triangle is 4 square units when the vertices are \((k,0),(4,0)\) & \((0,2)\)

4. If \( \tan^{-1} x + \tan^{-1} y = \frac{\pi}{4} \), then write the value of \( x + y + xy \).

5. Find a vector in the direction of \( \hat{i} - 2\hat{j} \), with a magnitude of 7 units.

6. If for a matrix \( A = \begin{bmatrix} a_{ij} \end{bmatrix} \), \( A_{ij} \) are the corresponding cofactors of \( a_{ij} \) then write the value of \( a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} \)

Section B

7. Solve the following differential equation : \( y e^y dx = \left( y^3 + 2xe^y \right) dy \), given that \( y = 1, \) when \( x = 0. \)

8. Find the differential equation of all circles in the first quadrant which touch the coordinate axes.
9. In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, One student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance? Also state the factors which affect the result of a student in the examination.

10. Prove that: \( \cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3 \).

11. Verify Rolle’s theorem for the function \( f(x) = x(x+3)e^{-\frac{x}{2}} \) on \([-3,0]\).

(or)

Find the points of local minima and local maxima and the corresponding local maximum and minimum values of the function \( f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11 \).

12. Let "*" be a binary operation on \( A = Q \times Q \) defined by \( (a,b) \ast (c,d) = (ac + b + ad) \), \( \forall (a,b),(c,d) \in A \). Is "*" commutative? Is it associative? Find the identity element with respect to "*", if it exists.

(or)

Let “X” be a non empty set and "*" be a binary operation on \( P(X) \), the power set of “X”, defined as \( A \ast B = A \cup B \), \( \forall A,B \in P(X) \). Is "*" commutative? Is it associative?

Find the identity element with respect to "*" on \( P(X) \). Also find all the invertible elements on \( P(X) \).

13. If \( x = e^t \cos t, y = e^t \sin t \), then find \( \frac{d^2 y}{dx^2} \).

14. Differentiate \( \tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}, \ -1 < x < 1 \) & \( x \neq 0 \) with respect to \( \cos^{-1} x \).

15. Evaluate \( \int_{0}^{2} (x^2 + x + 1) \, dx \) as a limit of sum.

(or)

Evaluate \( \int_{0}^{\pi/2} \frac{\sin x + \cos x}{9 + 16 \sin 2x} \, dx \).
16. Find the equation of the plane containing the line of intersection of
\[ \mathbf{r}.(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) - 4 = 0 \quad \& \quad \mathbf{r}.(2\mathbf{i} + \mathbf{j} - \mathbf{k}) + 5 = 0 \]
which is perpendicular to
\[ \mathbf{r}.(5\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}) + 8 = 0 \]
(or)
Find the shortest distance between the lines whose vector equations are given by
\[ \mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \lambda (2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) \]
and
\[ \mathbf{r} = (2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) + \mu (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) \]

17. Evaluate: \[ \int_{0}^{\pi} \log (\tan x + \cot x) \, dx \]

18. Evaluate: \[ \int \frac{(x-1)^2}{x^2 + 3x + 2} \, dx \]

19. Using properties of determinants, show that:
\[ \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9(a+b)b^2 \]

Section C

20. For keeping fit x people believe in morning walk, y people believe in Yoga practices and z people believe in attending Gym. The total no of people is 70. 20% of people who believe in morning walk suffer from disease, 30% of people who believe in yoga suffer from disease and 40% of people who believe in attending Gym suffer from disease. Total no of people suffering from disease is 24. Morning walk costs Rs 0, yoga costs Rs 500/month, Gym costs Rs 400/month for an individual. The total expenditure of the group per month is Rs 23000. Using Matrix method calculate the no of each type of people. Why is exercise important for health?

21. The contents of three urns are: Urn I: 7 white 3 black balls, Urn II: 4 white and 6 black balls and Urn III: 2 white and 8 black balls. One of the urns is chosen at random with a probability of 0.20, 0.60 and 0.20 respectively. From the chosen urn 2 balls are drawn at random without replacement. a) What is the probability that the balls are one white and one black. (b) If one white and one black balls are drawn, what is the probability that it came from Urn III.

22. A car parking company has 500 subscribers and collects fixed charges of Rs.300 per subscriber per month. The company proposes to increase the monthly subscription and it is believed that for every increase of Re.1, one subscriber will discontinue the service. What increase will bring maximum income of the company? What values are driven by this problem?
23. Find the image of the point \( i + 3j + 4k \) in the plane \( \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0 \)

(Or)

Find the equation of the perpendicular from the point \((3,-1,11)\) to the line \( \frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} \). Also find the foot of the perpendicular.

24. a) Let \( \vec{a} \) and \( \vec{b} \) be two vectors such that \( |\vec{a}| = 3 \) and \( |\vec{b}| = \frac{\sqrt{2}}{3} \). If \( \vec{a} \times \vec{b} \) is a unit vector. Then find the angle between \( \vec{a} \) and \( \vec{b} \).

b) Given \( \vec{a} = \hat{i} + \hat{j} + \hat{k} \) and \( \vec{b} = 6\hat{i} - 3\hat{j} - 6\hat{k} \), express \( \vec{b} \) as \( \vec{b}_1 + \vec{b}_2 \) such that \( \vec{b}_1 \) is parallel to \( \vec{a} \) and \( \vec{b}_2 \) is perpendicular to the vector \( \vec{a} \).

25. A firm manufactures chairs and tables each requiring 3 machines A, B and C. Production of one chair requires 4 hrs. on machine A, 1 hr. on B and 1 hr. on C. A table requires 1 hr. on machine A, 2 hrs. on machine B and 1 hr. on machine C. The profit got by selling one chair is Rs. 30 and the profit for a table is Rs. 50. The total time available per week on machine A is 24 hrs., on machine B is 16 hrs. and on C is 9 hrs. How many chairs and tables should be made per week so as to maximize the profit. Formulate the problem as LPP and solve it graphically.

26. Sketch the region given by \( \{ (x, y): x^2 \leq 6y \& x^2 + y^2 \leq 16 \} \) and using integration find the area.

(Or)

Using integration find the area of the region bounded by the triangle ABC, whose vertices are given by \((-1,1), \ (0,5) \ & \ (3,2)\) respectively.
Sample Paper 3

Time: 3hrs. 

General Instructions:

6. All questions are compulsory.
7. The question paper consists of 26 questions divided into three sections A, B and C.
   Section A contains 6 questions of 1 mark each, section B is of 13 questions of 4 marks each and section C is of 7 questions of 6 marks each.
8. All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
9. There is no overall choice. However an internal choice has been provided in four questions of 4 marks each and two questions of 6 mark each.
10. Use of calculators is not permitted. However, you may ask for Mathematical tables.
11. There are 4 printed sides in this paper.

SECTION A

Q1. If \( f : A \rightarrow A \) is a bijective function defined by \( f(x) = 2x^2 + 1 \), find \( f^{-1}(9) \).

Q2. Evaluate \( \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\left(\sqrt{3}\right) \).

Q3. Find \( A^{-1} \) if \( |A| = 5 \), where A is a square matrix of order3.

Q4. If \( A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \) satisfies \( A^2 = kA \).

Q5. If \( |A| = 15 \), find \( |A^{-1}| \), where A is a non singular matrix.

Q6. Find the vector projection of \( 2\hat{i} - \hat{j} + 2\hat{k} \) on \( -2\hat{i} + 6\hat{j} - 3\hat{k} \).

SECTION B

Q7. Define a binary operation * on the set \( \{1, 2, 3, 4, 5\} \) as \( a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases} \).

Show that 0 is the identity element for this operation and also find the inverse of the elements.

Q8. Solve \( \tan^{-1} 2x + \tan^{-1} 3x = \tan^{-1} 1 \).
Q9. Using properties of determinants show that
\[
\begin{vmatrix}
1 + a^2 - b^2 & 2ab & -2b \\
2ab & 1 - a^2 + b^2 & 2a \\
2b & -2a & 1 - a^2 - b^2
\end{vmatrix} = (1 + a^2 + b^2)^3
\]

Q10. Solve the differential equation \( \cos(x + y)dy = dx \).

Q11. Differentiate \( \sin^{-1}\left(\frac{2x}{1 + x^2}\right) \) w.r.t. \( \tan^{-1} x \)

Q12. If \( y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1 \), prove that \( \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \)

Q13. Evaluate \( \int_{-\pi}^{\pi} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \) OR \( \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx \)

Q14. Find the equations of normal(s) to the curve \( y = x^3 + 2x + 6 \) which are parallel to the line \( x + 14y + 4 = 0 \). OR Water is dripping out from a conical funnel at a uniform rate of 4 \( \text{cm}^3/\text{sec} \) through a tiny hole at the vertex in the bottom. When the slant height of water is 3 cm, find the rate of decrease of the slant height of the water cone given that the vertical angle of the funnel is 120°.

Q15. Find the particular solution of the differential equation \( x \frac{dy}{dx} + y = x \cos x + \sin x \), given that \( y\left(\frac{\pi}{4}\right) = 1 \).

Q16. Let \( \mathbf{a} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \), \( \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \), \( \mathbf{c} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} \). Find a vector \( \mathbf{d} \) which is perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \) such that \( \mathbf{c} \cdot \mathbf{d} = 15 \) OR

The scalar product of \( \mathbf{i} + \mathbf{j} + \mathbf{k} \) with a unit vector along the sum of \( 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} \) and \( \lambda\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \) is equal to 1. Find the value of \( \lambda \).

Q17. Evaluate \( \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx \)
Q18. Evaluate \( \int \frac{\tan \theta + \tan^3 \theta}{1 + \tan^3 \theta} \, d\theta \)

Q19. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is strike. Determine the probability that the construction job will be completed on time. What values are driven by this question?

SECTION C

Q20. Show that the semi vertical angle of a right circular cone of given surface area and maximum volume is \( \sin^{-1}\left(\frac{1}{3}\right) \).

Q21. A school has to reward the students participating in co-curricular activities (category 1), with 100% attendance (category 2) and brave students (category 3) in a function. The sum of the numbers of students of all the categories is 6. If we multiply the number belonging to category 3 by 2 and add to the number belonging to category 1, the result is 7. By adding the numbers belonging to category 1 and category 2 to 3 times that of category 3, the result is 12. Using Matrix Method, find the number of students belonging to three different categories.

Q22. Show that the points (0,-1,-1), (-4,4,4), (4,5,1) and (3,9,4) are coplanar. Find the equation of the plane in vector form and Cartesian form.

OR

Find the shortest distance between the lines whose equations are

\[ \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 4\hat{k}) \] and \[ \vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu (4\hat{i} + 6\hat{j} + 8\hat{k}) \]. Also find the equation of the plane containing them.

Q23. Find the area of the region \( \{(x,y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\} \)

OR

Find the area of the region bounded by the lines, using the method of integration.

\[ 2x + y = 4 \ , \ 3x - 2y = 6 \ , \ x - 3y + 5 = 0 \]
Q24. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the mean variance and standard deviation of the number of kings.

Q25. A dietician wishes to mix two types of food in such a way that the vitamin content of the mixture contains at least 8 unit of vitamin A and 10 unit of vitamin C. Food I contains 2 unit/kg of vitamin A and 1 unit/kg of vitamin C, while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs.5.00 per kg to purchase food I and Rs.7.00 per kg to produce food II. Determine the minimum cost of the mixture. Formulate the LPP and solve it. Why should a person take balanced diet?

Q26. Find the image of the point (1,2,3) in the line \[ \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} \]

OR

Find the distance of the point (1,-2,3) from the plane \[ x - y + z + 1 = 0 \] measured along the line parallel to \[ \frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \]
Assignment 1

1. $\sqrt{2}$

2. $e = 0$

4. $g \circ f(x) = 2x$
   \[ f \circ g(x) = 8x \]

5. $\sin 2x$

7. $f^{-1} : S \rightarrow S$
   \[ f^{-1} = \{(3,1), (2,3), (1,2)\} \]

11. $f \circ g$ does not exist

12. $[0,1)$

13. a) Many One ; Into
   b) One One ; Onto

16. 64

17. $\frac{5x + 2}{3}$

18. 2

19. 2

21. a) Many One
    b) Many One
    c) One One
    d) One One

22. Commutative & not associative.

23. Identity element is $(1,0)$;
   Inverse of $(a,b)$ is $\left(\frac{1}{a}, -\frac{b}{a}\right)$

24. $f^{-1}(x) = \frac{x}{1-x}$

25. $f^{-1}(x) = (x-27)^{\frac{1}{3}}$
   \[ f^{-1}(0) = -3 \]

Assignment 2:

1. $\frac{\pi}{2}$

2. $\frac{3\pi}{4}$

3. $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$  \[ 4.0 \]

5. $\frac{5}{12}$

6. $\frac{3}{5}$

7. $\frac{1}{2}$  \[ 8. \frac{\pi}{4} \]
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9. \( \frac{\sqrt{3}}{2} \)

11.a) \( \frac{1}{2} \tan^{-1} x \)

14. \( x = \frac{1}{4} \)

b) \( \frac{x}{2} \) / \( \frac{\pi}{2} - x \)

c) \( \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x \)

Assignment No. 3

1. \( x = 3; y = 3 \)

2. \( I \)

3. \( \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & 1 \\ \frac{5}{2} & 2 \end{bmatrix} \)

4. 0

5. \( \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix} \)

6. \( \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix} \)

7. \( \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 \\ -1/2 \\ -9/2 \end{bmatrix} \)

8. \( x = -1; x = -2 \)

10. \( \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix} \)

Assignment No. 4

1. 36

2. 25

3. 0

4. 64

5. -15

6. 216

7. a) \( x = 2, y = 3, z = 5 \)

7.b) \( x = 1, y = 2, z = 5 \)

8. \( x = 2, y = -1, z = 4 \)

9. \( x = 1, y = 1, z = 1 \)

10. \( \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix} \)

11. \( \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \)

12. \( k = -6 \) or 14
Assignment No. 5(a)

1. \(\frac{1}{2}x^2\)  
2. \(-\frac{x^4}{y^5}\)  
4. \(\frac{2}{1 + x^2}\)  
5. Discontinuous at \(x = -2, -1, 0, 1, 2\)  
6. \(\frac{1}{2(1 + x^2)}\)  
8. \(\frac{1 + x^2}{2}\)  
9. 5  
10. \(a^{(\sin^{-1}x)^2}\frac{2\log a}{\sqrt{1 - x^2}}\)  
11. \(\frac{1}{2}\sqrt{12}\)  
12. \(a = -\frac{3}{2}, \ c = \frac{1}{2}\) & \(b \in \mathbb{R}\)  
13. Not derivable  
14. \(a = 2, \ b = -1\)  
15. Continuous

Assignment No. 5 (b)

1. \(x = \pi\)  
2. \(x^{\sin x} \left[\frac{\sin x}{x} + \cos x \log x\right] + \frac{1}{\sqrt{1-x^2}} \frac{1}{2\sqrt{x}}\)  
11. \(-\frac{ax + hy + g}{hx + by + f}\)  
13. \(x^5\)

Assignment No. 6

1. \(\sqrt{\frac{2}{4\pi}}\) cm/sec  
2. \(-\frac{2}{75}\) radian/sec  
3. 5.02, 3.074, 0.1925  
4. \((-1, \infty)\) function is increasing.  
5. \((-\infty, -1) \cup (-1, 1)\) function decreasing , \((1, 3)\) & \((3, \infty)\) function increases ; point of minima is 1, points of inflexion are -1, 3.  
6. \(x = 3\) point of maxima, \(x = 0, 5\) are points of minima. Minimum values : \(f(0) = 105; \ f(5) = \frac{545}{4}\); Maximum value: \(f(3) = \frac{609}{4}\).

7. \((4, -4)\)  
8. \(y + 3x = 3\) & \(y = 7x - 14\)  
9. Increasing in the interval \((-\infty, 1) \cup (2, \infty)\) and decreasing in the interval \((1, 2)\)  
10. \(\left(0, \frac{\pi}{4}\right)\) decreasing and \(\left(\frac{\pi}{4}, \frac{\pi}{2}\right)\) increasing  
11.a) \(x = \frac{\pi}{6}\) is a point of maxima and the maximum value \(\sqrt{3} + \frac{\pi}{6}\). Point of minima is \(x = \frac{5\pi}{6}\).  

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and the minimum value is $-\sqrt{3} + \frac{5\pi}{6}$.

11(b) $x = \frac{\pi}{3}$ is a point of maxima and the maximum value is $\sqrt{3} - \frac{\pi}{3}$; $x = -\frac{\pi}{3}$ is a point of minima and the minimum value is $-\sqrt{3} + \frac{\pi}{3}$.

12. $y = \frac{\pi}{4} + \frac{1}{\sqrt{2}}(\sqrt{2} - 1)\left(x - \frac{\sqrt{2} - 1}{\sqrt{2}}\right)$ & $y = \frac{\pi}{4} + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2} - 1}\left(x - \frac{\sqrt{2} - 1}{\sqrt{2}}\right)$

Assignment No. 7 (a)

1. $\sqrt{x^2 + 1} - 2\log|x + \sqrt{x^2 + 1}| + c$
2. $\frac{2}{3} x^3 + \frac{x^3}{3} + c$
3. $\frac{1}{2} \log|\cos ec(x + \frac{\pi}{6}) - \cot(x + \frac{\pi}{6})|$

4. $\log|x + \log x|$
5. 8
6. $x \sin(\log x)$
7. $k = \frac{1}{\log 2}$
8. $\frac{1}{24} \log \frac{3x - 4}{3x + 4}$

9. $\frac{1}{2} e^{-x^2} + c$
10. $\tan^{-1} e^x + c$
11. $\frac{1}{3} \log \frac{2\tan x + 1}{\tan x - 2} + C$
12. $-\sqrt{1 + 2 \cot x} + c$

13. $\cot x + \frac{1}{7} \cot^2 x + \frac{3}{5} \cot^5 x + \cot^3 x + c$
14. $\frac{1}{(\log 5)^3} 5^{5x} + c$

15. $\frac{1}{3} \log \left|x^3 + \sqrt{x^6 - a^6}\right| + c$
16. $\frac{x^2}{2} + 2x + \frac{3}{2} \log|x^2 - x + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x - 1}{\sqrt{3}} + c$

17. $\sqrt{-x^2 + 5x + 6} + \frac{1}{2} \sin^{-1} (2x - 5) + c$
18. $\log \left|\frac{2e^x + 1}{2e^x + 2}\right| + c$

19. $-3\sqrt{6 + x - 2x^2} + \frac{13}{2\sqrt{2}} \sin^{-1} \frac{4x - 1}{7} + c$
20. $\sin^{-1} (2\sin x - 1) + c$

21. $\frac{-1}{3(\tan x + 1)} + C$
22. $\log(\log(\log x)) + C$
23. $\sqrt{2} \sin^{-1}(\sin \theta - \cos \theta) + C$

24. $\frac{1}{2} \log \left|\frac{\tan x}{\tan x + 2}\right| + C$
25. $\log x^2 + C$
26. $\log \left|\cos x + \frac{1}{2} + \sqrt{\cos^2 x + \cos x}\right| + C$
Assignment 7 (b)

1. \( \frac{1}{4} \log |x - 1| + \frac{3}{4} \log |x + 1| + \frac{1}{2(x+1)} + c \)

2. \( 3 \left[ \cos^{-1} x - \sqrt{1-x^2} \right] + c \)

3. \( \frac{e^x}{(x+1)^2} + c \)

4. \( -\frac{1}{3} \log |\tan x + 1| + \frac{1}{6} \log |\tan^2 x - \tan x + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x - 1}{\sqrt{3}} + c \)

5. \( e^{-x} \sec \frac{x}{2} + c \)

6. \( -\tan^{-1} \frac{x}{\sqrt{1 + x^2}} + c \)

7. \( -\log \left( \frac{x+2}{x+2} \right) - \frac{1}{x+2} + c \)

8. \( -\frac{1}{2} \log |\cos x| + \frac{1}{18} \log |\cos x| + \frac{4}{9} \log |5 - 4 \cos x| + c \)

9. \( 2 \left[ \sqrt{x} e^{\sqrt{x}} - e^{\sqrt{x}} \right] + c \)

10. \( \frac{3}{2} \sqrt{x} \sin \sqrt{x} + \frac{3}{2} \cos \sqrt{x} + \frac{1}{6} \sqrt{x} \sin 3\sqrt{x} + \frac{1}{18} \cos 3\sqrt{x} + c \)

11. \( \frac{9}{13} \left[ -\frac{1}{3} e^{-3x} \cos 2x + \frac{2}{9} e^{-3x} \sin 2x \right] + c \)

12. \( e^x \left[ -\log \cos x \right] + c \)

13. \( \frac{1}{2} e^{2x} \cot 2x + c \)

14. \( x - \log x + \frac{1}{2} \log |x^2 + 1| - \tan^{-1} x + c \)

15. \( -\frac{1}{3} \left( 3 - x - x^2 \right)^2 + \frac{1}{2} \left[ \frac{2x+1}{4} \sqrt{3-x-x^2} + \frac{13}{8} \sin^{-1} \frac{2x+1}{\sqrt{13}} \right] + c \)

16. \( \frac{x^3}{3} \cos ec^{-1} x + \frac{1}{3} \left[ \frac{x}{2} \sqrt{x^2 - 1} - \frac{1}{2} \log |x + \sqrt{x^2 - 1}| \right] + \frac{1}{3} \log |x + \sqrt{x^2 - 1}| + c \)

17. \( x \log (\log x) - \frac{x}{\log x} + c \)

18. \( \left( x^2 + 2x - 3 \right)^3 - 2 \left[ \frac{x+1}{2} \sqrt{x^2 + 2x - 3} - 2 \log |x + 1 + \sqrt{x^2 + 2x - 3}| \right] + c \)
Assignment 8

1. 0.5  2. –2  3.2
4. 2  5. e – 1
6. \( \frac{1}{2} \)  7. \( \frac{\pi}{4} \)
8. \( a = -1, b = 1 \)  9. (i) \(-\log 2\)  (ii) \(\sqrt{2}\pi\)

(iii) \(\frac{\pi}{\sqrt{35}}\)  (iv) \(\frac{\pi}{12}\)  (v) 0  (vi) \(\frac{\pi^2}{16}\)  (vii) \(\frac{\pi}{2} - \log 2\)

(viii) \(\frac{\pi}{8}\) \(\log 2\)  (ix) 2  (x) \(\frac{\pi}{2}\)  (xi) \(\frac{63}{2}\)

10. (i) \(\frac{-7}{6}\)  (ii) \(\frac{e^2 (e^4 - 1)}{2}\)  (iii) 32

(iv) \(e^4 + 7\)

Assignment 9

1. \(\left(16 - \frac{4}{3} \sqrt{2}\right)\) sq units  2. \(\frac{4\sqrt{3} + 16\pi}{3}\) sq units

3. \(\frac{16a^2}{3}\) sq units  4. 7 sq units  5. 6 sq units

6. \(\frac{4}{3}\) sq units  7. \((\pi)\) sq units

8. 10 sq units  9. \(\frac{6\pi}{7}\) sq units  10. \(\frac{9\pi}{7}\) sq units

11. \(\left(\frac{\pi}{4} - \frac{1}{2}\right)\) sq units  12. \(\frac{5\pi}{2} - \sin^{-1} \frac{1}{4} - 4\sin^{-1} \frac{7}{8} - \frac{\sqrt{15}}{2}\) sq units
Assignment 10

1. \(\frac{d^2y}{dx^2} - y = 0\) 2. \(y = x\) 5. (i) 2, 4 (ii) 2, n.d.
(iii) 1, 2 (iv) 2, 3

6. (i) \(e^{\tan^{-1}x}\) (ii) \(\frac{1}{x}\) (iii) \(\frac{1}{y^2}\) (iv) \(e^{2x}\) 7. (i) \(\log |y| = 2x + 2 \log |x-1| - 4\)
(ii) \(\log |x| + x = -\log |y| - y + c\) (iii) \(e^{2x}y = -\cos x + 1\)
(iv) \(x = 2y^2\) (v) \(\log \left| 1 + \tan \frac{x+y}{2} \right| = x + c\)
(vi) \(c - \sqrt{1+y^2} = \sqrt{1+x^2} + \frac{1}{2} \log \left| \frac{1+x^2-1}{1+x^2+1} \right|\)
(vii) \(x = y + 1 - \log |x + y + 2| = x + c\)
(viii) \(x^2 \sin x + 2x \cos x - 2 \sin x - x^2 y + c = 0\)
(ix) \(-\tan^{-1} y = \tan^{-1} e^x - \frac{\pi}{2}\)
(x) \(\log |1 + y| = x - \frac{x^2}{2} + c\)

Assignment 11 (a)

1. \(\frac{1}{2} i + \frac{\sqrt{3}}{2} j\) 2. \(\sqrt{a^2 + b^2}\) 3. \(i + j, -i - j\)
4. 5 5. \(\frac{3}{7}, -\frac{6}{7}, \frac{2}{7}\)

6. \(\pm 6\) 7. \(\frac{\pi}{3}, \frac{2\pi}{3}\) 8. \(\pm 2\sqrt{3}\) 9. \(\frac{9i + 11j + 15k}{3}\)

10. (i) \(\cos^{-1} \frac{1}{\sqrt{3}}, \cos^{-1} \frac{1}{\sqrt{3}}, \cos^{-1} \frac{1}{\sqrt{3}}\)
(ii) \(\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\) 12. \(-2i + j + 5k\) \(\frac{13.2i - 2j + k}{\sqrt{30}}\)
14. \(\sqrt{114}\) 15. \(\frac{5}{\sqrt{6}}, \frac{5}{6}(i - 2j + k)\)
17. \(-3\) 18. \(x = -6, y = 2\)
19. \(4i + j\) 20. \(\pm 6\) 21. \(90^\circ\) 22. 21
Assignment 11(b)

1. 2, -1  
2. \( \frac{\pi}{4} \)  
3. \( 3\sqrt{2} \)  
4. \( \frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}, \frac{\hat{i} - \hat{k}}{\sqrt{2}} \)  
5. \( 5\sqrt{17} \text{ sq units} \)  
6. \( 3 \frac{27}{2} \)  
7. \( \hat{i} + 3\hat{j} + 3\hat{k} \)  
8. \( \hat{i} + \hat{j} + \hat{k} \)  
9. \( 264 \text{ cubic units} \)  
10. \( \lambda = 1 \)  
11. \( \alpha = -2, -3 \)  
12. -1  
13. \( \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}, \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} \)  
14. \( \pm \frac{100}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) \)

Assignment 12

1. \( \frac{9}{\sqrt{50}} \)  
2. \( \vec{r} \cdot \hat{j} = 3 \)  
3. \( 3.3x + 2y - z - 3 = 0 \)  
4. \( -\frac{10}{7} \)  
5. \( 5\vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k} + \mu(2\hat{i} + 5\hat{k}) \)  
6. \( \frac{9\hat{i} + 11\hat{j} + 15\hat{k}}{3} \)  
7. \( \vec{r} = \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k} + \lambda\left( \frac{1}{3}\hat{i} + \frac{1}{6}\hat{j} - \hat{k} \right) \)  
8. \( \text{acute angle} = \sin^{-1} \frac{1}{3\sqrt{2}} \)  
9. \( 9, \frac{7}{3}, \frac{7}{9} \)  
10. \( z = 5 \)  
11. \( (3, 5, 9), -18x + 22y - 5z = 11 \)  
12. \( \vec{r} \cdot (7\hat{i} + 9\hat{j} - 10\hat{k}) = 27 \)  
13. \( \vec{r} \cdot (4\hat{i} - \hat{j} - 2\hat{k}) = 6 \)  
14. \( (a)\sqrt{62} \text{ units} \)  
15. \( \vec{r} \cdot (18\hat{i} + 17\hat{j} + 4\hat{k}) = 49 \)  
16. \( \vec{r} \cdot (\hat{i} + 3\hat{j} - 4\hat{k}) = -6 \)  
17. \( \vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) = -6 \)  
18. \( 5y - 5z - 6 = 0, -\frac{6}{\sqrt{5}} \text{ units} \)  
19. \( 21.6 \text{ units} \)  
20. \( 22. \frac{10}{3\sqrt{3}} \text{ units} \)  
21. \( \left( \frac{17}{3}, 0, \frac{23}{3} \right) \)  
22. \( 24x - 2y + 2z + 2 = 0, x - 2y + 2z - 4 = 0 \)
Assignment 13

1. \( \frac{3}{8}, 2.09, \frac{3}{5}, \frac{4}{2}, 5.0.12 \)
2. \( \frac{15}{17}, \frac{8.29}{45} \)
3. \( \frac{3}{8}, 10.\frac{11}{21} \)
4. \( \frac{293}{630}, \frac{90}{293}, \frac{12}{9}, 13.\frac{12}{17}, \frac{5}{17} \)

11. (a) \( \frac{293}{630} \), (b) \( \frac{90}{293} \), (i) \( \frac{12}{17} \), (ii) \( \frac{5}{17} \)

14. Mean = \( \frac{2}{5} \), Variance = \( \frac{216}{475} \), SD = \( \frac{6}{5} \sqrt{\frac{6}{19}} \)

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<td>P(X)</td>
<td>12/19</td>
<td>32/95</td>
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15. Mean = \( \frac{4}{3} \), SD = \( \frac{\sqrt{5}}{3} \)

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<td>P(X)</td>
<td>5/42</td>
<td>20/42</td>
<td>15/42</td>
<td>2/42</td>
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16. (i) \( \frac{19}{144} \) (ii) \( \frac{1175}{64} \) (iii) \( \frac{7}{432} \) (iv) \( \frac{1125}{64} \)

Answers to Practice Tests

Practice Test 1

2. Not injective
3. \{ (1,5), (2,11), (8,0) \}
4. I
5. \( \frac{11}{27} \)
6. Not associative
7. \( \frac{\sqrt{x+6}-1}{3} \)
10. \( \frac{2x-1}{3} \)
11. \( \frac{7y+4}{5y-3} \)
12. It is commutative and associative, e=0

Practice Test 2

1. \( \frac{5}{12} \)
2. \( \frac{7\pi}{6} \)
3. \( \frac{7\pi}{12} \)
4. \( \frac{\sqrt{3}}{2} \)
5. \( \frac{7\pi}{12} \)
6. \( \frac{\pi}{6} \)
7. \( \frac{1}{4} \)
11. \( \frac{1}{6} \)

Practice Test 3

1. \( x = 2 \)
2. \( B = \begin{bmatrix} -5 & -4 \\ -5 & -6 \end{bmatrix} \)
3. \( = 8 \)
4. \( -256 \)
7. \( \begin{bmatrix} 2/3 & -1/6 \\ -1 & 1/2 \end{bmatrix} \)
10. 300, 100, 200
Practice Test 4

1. \( \lambda = -4 \)  
2. 1  
4. \( a = \frac{1}{2} \)

5. \( x^{\cos x} [\cos x - x \log x + \cos x \log x] - \frac{4x}{(x^2 - 1)^2} \)

6. \( \frac{y (x \log y - x)}{x (y \log x - x)} \)
7. \( \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}} \)

8. \( \frac{-1}{\sqrt{1-x^2}} \)
9. \( \frac{8b\pi}{3a^2} \)
10. \( 1 - \frac{\sqrt{21}}{6} \)  
12. 2

Practice Test 5

1. 0.0608  
2. .03x^3 cm^3  
3. Increasing in \((-\infty, 1) \cup \left(1, \frac{8}{5}\right) \cup (2, \infty)\), decreasing in \(\left[\frac{8}{5}, 2\right]\)

4. \( \frac{y - b \cos^3 t}{x - a \sin^3 t} = -\frac{b}{a} \cot t \) and \( \frac{y - b \cos^3 t}{x - a \sin^3 t} = \frac{a}{b} \tan t \)

5. \( y - \left(2 + 2\sqrt{3}\right) = 2\sqrt{3}(x - 2) \) and \( y - \left(2 - 2\sqrt{3}\right) = -2\sqrt{3}(x - 2) \)

6. Increasing in \((-\infty, 1) \cup (1, 3)\) and decreasing in \((3, \infty)\)

7. local max. at -5 and 0, local min. at -3

8. local max. at \(\frac{\pi}{2}\) and local min. at \(\frac{\pi}{6}\)

9. \( \frac{-32}{27\pi} \)  
10. \( \frac{1}{2\pi} \) cm/s

Practice Test 6

\( Q1. x + 2 \log \left| \frac{x - 1}{x - 2} \right| + c \)

\( Q2. \frac{1}{8} \log |1 - \sin x| - \frac{1}{8} \log |1 + \sin x| - \frac{1}{4\sqrt{2}} \log |1 - \sqrt{2} \sin x| + \frac{1}{4\sqrt{2}} \log |1 + \sqrt{2} \sin x| + c \)

\( Q3. \frac{1}{4} e^{2x} + \frac{1}{8} e^{2x} (\sin 2x + \cos 2x) + c \)

\( Q4. \frac{-2}{5} \tan^{-1} x + \frac{3}{5} \tan^{-1} x + c \)
\[ Q6. \frac{-1}{3}(1+x-x^2)^{3/2} + \frac{1}{2} \left[ \frac{1}{2} \left( x - \frac{1}{2} \right) \sqrt{1+x-x^2} + \frac{5}{8} \sin^{-1}\left( \frac{2x-1}{\sqrt{5}} \right) \right] + c \]

\[ Q7. \frac{1}{5} \log \left| \frac{\tan^2 \frac{x}{2} + 2}{\tan^2 \frac{x}{2} - 3} \right| + c \]

\[ Q8. \pi a \quad Q9. \frac{-1}{\sqrt{2}} \log(\sqrt{2} - 1) \quad Q10. 3 \]

\[ Q11. \frac{1}{18} \log |l + \sin x| - \frac{1}{2} \log |l - \sin x| + \frac{4}{9} \log |5 - 4 \sin x| + c \]

**Practice Test 7**

\[ Q1. (x^2 - y^2) \frac{dy}{dx} - 2xy = 0 \quad Q2. (x - y)^2 \left( 1 + \left( \frac{dy}{dx} \right)^2 \right) = (x + y \frac{dy}{dx})^2 \]

\[ Q3. y = (x + 1) \log |x + 1| - x + 5 \quad Q4. cx = ye^{\frac{1}{x}} \]

\[ Q5. y = c + a \tan^{-1} \left( \frac{x + y}{a} \right) \quad Q6. x^4 + 6x^2y^2 + y^4 = 8 \quad Q7. xe^{\tan^{-1}y} = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c \]

\[ Q8. x = y^2 (e^{-1} - e^{-y}) \quad Q9. \left( \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} \right) \text{ sq. units} \quad Q10. \frac{11}{6} \quad Q11. \frac{4\sqrt{3}}{3} + \frac{16\pi}{3} \text{ sq. units} \]

**Practice Test 8**

\[ Q1.2\hat{i} - 4\hat{j} + 4\hat{k} \quad Q3. 5x + 9y + 4z - 37 = 0 \quad Q4.(-4,-1,-3) \quad Q5.15\hat{i} - 27\hat{j} + 5\hat{k} \]

\[ Q6. \vec{a} = \frac{1}{4} \hat{i} + \frac{1}{4} \hat{j} + \frac{3}{4} \hat{k} \quad Q8.1, -2, 7) \quad Q9. \vec{r} \left( 33\hat{i} + 45\hat{j} + 50\hat{k} \right) - 41 = 0 \quad Q10. \text{ distance } = 13 \]

\[ Q11. \text{ Foot of perpendicular is } (1,3,0), \text{ perpendicular distance } = \sqrt{6}, \text{ image is } (-1,4,1) \]

**Practice Test 9**

\[ Q1. \frac{1}{3} \quad Q2. (i) \frac{11}{12} \quad (ii) \frac{5}{12} \quad (iii) \frac{7}{8} \quad Q3. \frac{307}{686} \quad Q4. \frac{5}{11} \quad Q5. \frac{18}{143} \quad Q6. \frac{3}{13} \quad Q7. \frac{11}{50} \]

\[ Q8. \text{ no. of trials } \geq 4 \quad Q9. \frac{3}{4} \quad Q10. \text{ 0, he neither loses nor wins} \quad Q11. \text{ Mean } = 2, \text{ variance } = 1 \]