Assignment No. 6

Application of Derivatives

1. Water is dripping out from a conical funnel of semi vertical angle $\frac{\pi}{4}$ at a uniform speed of 2 $cm^3/\sec$ through a tiny hole at the vertex of the bottom. When the slant height of water is 4cm , find the rate of decrease of slant height of the water.
2. A man is moving away from a tower 49.6 m high at the rate of 2 m/s. Find the rate at which the angle of elevation of the top of the tower is changing, when he is at a distance of 36 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.
3. Evaluate following up to three decimal places using differentiation:
   $\sqrt{25.2}$, $\sqrt[3]{29}$, $\sqrt{0.037}$
4. Find the intervals in which the function $f(x)=\log(1+x)-\frac{2x}{2+x}$ increasing or decreasing.
5. Find the intervals in which the function $f(x)=(x+1)^3(x-3)^3$ is increasing or decreasing. Also find the points at which the function has local maxima, local minima and the point of inflexion.
6. Find all the points of local maximum and minimum and the corresponding maximum and minimum values of the following function $\frac{3}{4}x^4-8x^3+\frac{45}{2}x^2+105$.
7. Find the point on the curve $y^2=4x$ which is nearest to the point $(2,-8)$
8. Find the equation of the tangent to the curve $y=(x^3-1)(x-2)$ at the points where the curve cuts the x –axis.
9. Find the intervals in which the function $f(x)=2x^3-9x^2+12x+15$ is increasing and decreasing.
10. Separate $\left[0,\frac{\pi}{2}\right]$ into sub intervals in which $f(x)=\sin^4 x+\cos^4 x$ is increasing or decreasing.

11. Find the points of local maxima and local minima and also the local maximum and local minimum values of the following functions : $(i)\, f(x) = 2\cos x + x, x \in (0, \pi)$ $(ii)\, f(x) = 2\sin x - x, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
12. Find the equation of the tangent and normal to the curve $x = 1 - \cos\theta;\ y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$
13. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of cone.
14. An open box with a square base is to be made of given iron sheet of area 27 sq.m. Show that the maximum volume of the box is 13.5 cu. m.
15. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.